Lectures in International Finance

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1.1 Introduction

In a world where consumption, production, investment and capital markets are globalized, international finance, or international monetary economics has become an integral part of any study of international economics.

When studying international finance which is, broadly speaking, concerned with the monetary and macroeconomic relations between countries, a number of special problems arise. Many of these problems are due to the use of different currencies in different countries and the consequent need to exchange them. Rates of exchange between currencies are set by a variety of arrangements and both rates and arrangements are subject to change. Furthermore, exchange-rate changes can have sizable effects on balance-of-payments, economic activity, and policy strategy of the countries involved.

In this chapter, we look at some preliminary issues. We examine the operation of the key market without which no foreign transaction is possible: the foreign exchange market. We introduce the terminology used in foreign markets and discuss the basic structure and forces that operate in the market. We then examine the different type of transactions and the main financial instruments that are managed in currency markets.

1.2 The Markets for Foreign Exchange

The foreign exchange, or *forex* (FX) market is the market where exchange rates are determined. An exchange rate is a price, specifically the price of one currency in terms of another. It is the mechanism by which world currencies are tied together in the global marketplace. There are two common ways of expressing it. One, also referred to as *price quotation system* or *direct quotation*, expresses the exchange rate as the price of foreign currency in terms of domestic currency, i.e., as the amount of domestic currency needed to purchase one unit of a foreign currency. The other, also known as *volume quotation system* or *indirect quotation*, expresses the exchange rate as the price of domestic currency in terms of foreign currency, i.e., as the amount of foreign currency required to purchase one unit of domestic currency. Note that the indirect quotation is just the reciprocal of the corresponding direct quotation.[1] In this book we will use the former quotation. Hence, unless otherwise stated, we define the exchange rate as the price of domestic currency in terms of foreign currency. This implies that a rise (decrease) in the exchange rate means a depreciation (appreciation) of the domestic currency. For example, a rise in the dollars per euro from \$1.101/€1 to \$1.202/€1, means that the dollar has depreciated in value, whereas a decrease from \$1.101/€1 to \$0.998/€1 means that the dollar appreciated in value. The opposite obviously holds if we used the indirect quotation.

For the purpose of this chapter we refer to the foreign exchange, or currency market as a single worldwide market where the currencies of different countries are exchanged or traded. However, its major operators buy and sell foreign exchange from computer terminals which are

[1]Most currencies in the world are stated in terms of the number of units of foreign currency needed to buy one dollar. This quote, called "European" quote, expresses the rate as the foreign currency price of one U.S. dollar. For example, the exchange rate between U.S. dollars and Japanese yen is normally stated:

$$\text{JPY}122.816/\$1,$$

read as "122.816 Japanese yen per dollar". An alternative method, called" American" quote, states foreign exchange rates as the U.S. dollar price of one unit of foreign currency. The same exchange rate above expressed in American terms is:

$$\$0.00814/\text{JPY}1,$$

read as "0.00814 dollars per Japanese yen". Note that European quote and American quote are reciprocals:

$$\frac{1}{\text{JPY}122.816/\$1} = \$0.00814/\text{JPY}1.$$

physically located all around the world. The foreign exchange market is open 24 hours a day, split over three time zones. Trading begins each day in Sydney, and moves around the world as the business day begins in each financial center, first to Tokyo, then to London, and New York. The foreign exchange market has no physical venue where traders meet to deal in currencies. Computer screens around the world do the job, continuously showing exchange rate prices to traders so that they can deal with each other and find people willing to meet that price. It does not matter where the counterparties are located, if in London, Singapore, New York, Tokyo, Zurich, or Frankfurt. What matter is simply the willing to meet that price. There are, therefore, distinct FX markets in the real world, although they are linked by *arbitrage*, a mechanism ensuring that the rate of exchange between currencies quoted in different markets (i.e., in London, Tokyo, and other international financial center) be the same. The process of arbitrage is discussed below and justify the convention we followed here to talk about the forex market as a single market covering the whole world.

1.3 Characteristics, Activities and Players

The forex market is the most liquid and largest financial market in the world. According to the Triennial Central Bank Survey carried out by the Bank for International Settlements (BIS), the trading in foreign exchange markets averaged \$5.3 trillion per day in April 2013.[2] The daily average volume was about ten times the daily volume of all the world's equity markets and sixty times the U.S. daily GDP. The US dollar remained the dominant vehicle currency: it was involved in 87% of all trades in April 2013. The euro was the second most traded currency (33%), followed by the Japanese yen (23%), and the British pound (12%). Several emerging market currencies, and the Mexican peso and Chinese renminbi entered the list of the top 10 most traded currencies.

Trading was increasingly concentrated in the largest financial centres. In April 2013, sales desks in the United Kingdom, the United States, Singapore and Japan intermediated 71% of foreign exchange trading, whereas the major markets in the European Union (Frankfurt, Paris, and Amsterdam) played a smaller role.

Given the international nature of the market, the majority (57%)

[2] See BIS (2013).

of all foreign exchange transactions involved cross-border counterparties. This highlights one of the main concerns in the foreign exchange market: counterparty risk.

Foreign exchange is an over-the-counter (OTC) market where brokers/dealers negotiate directly with one another, so there is no central exchange or clearing house.

The main activities in the FX markets are speculation and arbitrage, foreign assets and international trade financing, hedging.

Speculation and arbitrage

Speculation and arbitrage are both financial strategies used by traders, to make larger profits. However, the technique in which each strategy is used differs substantially.

In a typical speculation activity, a trader takes on a foreign exchange position on the expectation of a favorable currency rate change. For example, a trader may open a *long position* on a currency (i.e., buy the currency today) with the expectation of profiting from a future appreciation (i.e., sell the currency at a higher price in the future). If the currency appreciates, the trader may close the trade for a profit. Conversely, if the currency depreciates, the trade might be closed for a loss. A speculator who expects a currency to appreciate in the future is said to be *bullish*.

Speculators, on the other hand, may open a *short position* (i.e., selling short, or simply selling the currency today in the hope of buying it back at a lower rate in the future) if they expect the currency to depreciate. If the currency drop, the position will be profitable. If the currency rise, the trade may be closed at a loss. A speculator who expects a currency to depreciate in the future is said to be *bearish.*

Speculation, therefore, is a type of financial strategy that involves a significant amount of risk.

Arbitrage, on the other hand, is a riskless trading strategy that allows traders to take advantage of price differentials prevailing simultaneously in different markets and across currencies. The simplest form of arbitrage in the FX market is *spatial arbitrage*, which takes advantage of the pricing discrepancies across geographically separate markets. For example, if the dollar-euro exchange rate quoted in New York is \$1.101/€1 but \$1.085/€1 in London, it would pay the traders to buy euros in London and simultaneously sell them in New York, making a risk-free profit of 1.6 cents on every euro bought and sold. This activity will make the euro to appreciate in London and to depreciate in New York, leading to an arbitrage-free equilibrium where the rate quoted in the two centers are equal.[3] *Cross-currency*, or *triangular arbitrage*

[3] Notice that all the examples discussed here imply zero transaction costs. These

takes advantage of discrepancies in the cross rates of different currency pairs. To illustrate suppose that the dollar-euro rate is \$1.085/€1, and the pound-euro rate is £0.695/€1. The implied outright rate between the pound and the dollar is thus \$1.561/£1=(\$1.085/€1)/(£0.695/€1). If the actual pound-euro rate is instead £0.682/€1, implying a rate of \$1.591/£1=(\$1.085/€1)/(£0.682/€1), then a dealer wanting dollars would do better to first sell euros for pound, e.g., sell €1,000×0.682=£682, and then sell pound for dollars obtaining \$1,085.062=£682×1.591 for an arbitrage profit of \$1,085.062-\$1,000=\$85.062. *Covered interest arbitrage* takes advantage of a misalignment of spot and forward rates (which you will learn about in the next section), and domestic and foreign interest rates.

Foreign assets and international trade financing

Foreign assets and international trade financing is overwhelmingly the major business of forex markets. It allows individuals and firms conducting international business (e.g., importers and exporters, companies making direct foreign investments, international investors buying or selling debt or equity investments for their portfolios) to transfer purchasing power from one country and its currency to another, alter the structure of assets and liabilities in different countries, obtain or provide credit for international economic transactions.

Hedging

Hedging is a way used by companies, financial investors and other economic agents to insure against the foreign exchange risk resulting in all international transactions. Hence, by contrast to speculation, hedging is the activity of covering an open position. By using this strategy properly, a trader who is long on a particular foreign currency (orders to buy or book a fixed amount of a foreign currency) can be protected from downside risk (a downward price move), while a trader who is short on a particular foreign currency (orders to sell a fixed

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are costs incurred when buying or selling currencies. Transaction costs include brokers/dealers commissions and spreads. Spread, or *bid-ask spread* is the difference between the buying and selling price of a given currency which exists at the same moment in the forex markets. It represents the gross profit margin of the broker/dealer. In quotations on the foreign markets bids always precede asks. For example, if the quote for the dollar-euro rate is $1.08278 - 1.08313$, 1.08278 is the bid, or buying price (i.e., the rate at which a trader will purchase the dollar) and 1.08313 is the ask, or selling price (i.e., the rate at which a trader will sell the dollar). Hence, in this case the bid-ask spread is $1.08313 - 1.08278 = 0.00035$, and the gross profit margin of the broker/dealer is $0,032\% = (0.00035/1.08313) \times 100$ of the traded amount of dollars.

amount of a foreign currency) can be protected against upside risk (a upward price move).

The main players in the market are speculators and arbitragers, commercial and investment banks, foreign exchange brokers, retail clients, and central banks.

Speculators and arbitragers

Speculators and arbitragers are businesses, international investors, multinational corporations and other who seek to profit from trading in the market.

Commercial and investment banks

These are leading players who buy and sell currencies from each other within what is known as the interbank market. Commercial and investment banks not only trade for their customers and on their own behalf through proprietary desks (proprietary trading), but also provide the channel through which all other market participants must trade. They account for by far the largest proportion of total trading volume, thus playing the vital role of catalysts in international financial markets.

Foreign exchange brokers

Forex brokers are intermediaries that make it easier to connect traders in the interbank market. Typically, a forex broker offer quotations for most currency from the banks that they have relationships with, showing the best rates. There is a small cost a trader is likely to incur when dealing through a broker. This is known as a brokerage fee, which is a commission charged by brokers for every currency pair they offer on their trading platform.

Retail clients

These are made up of consumers and travellers, businesses, investors and others who need foreign currencies for their personal and business purposes. They normally do not buy and sell currencies directly with one another, rather they operate with brokers and commercial banks.

Central banks

These are institutions to which the management of exchange rates and foreign reserves (see, Section) is attributed. Central banks play a very important role in foreign exchange market. However, they normally do not undertake a significant volume of trading. They enter the FX markets to trade with other central banks, various international institutions, and to maintain financial stability in the exchange rate.

Chapter 2

The Exchange Rates

2.1 Introduction

In the previous section we referred to 'the exchange rate' as the relative price of two currencies. Yet, the use of the singular noun is a simplification, which disguises the fact that in reality a variety of exchange rates exists at the same moment between the same two currencies. There are exchange rates for cash, or bank notes; exchange rates for checks and for electronic payments (credit card); exchange rates for the purchase or sale of a foreign currency. These rates can differ because of transaction costs, carrying (storage and custody) costs, bid-ask spreads, brokerage fees. The differences are however very small, so that henceforth we shall argue as if there is only one exchange rate for each foreign currency. Nevertheless, there is still a set of rates for each currency which we cannot ignore.

We start by looking at the difference between the *spot* and *forward* exchange rates.

2.2 The Spot Exchange Rate

The spot, or current exchange rate is the rate paid for immediate delivery of a currency. Except for certain cases such as an exchange of banknotes, normally "immediate delivery" means settlement of the foreign-exchange contract within two business days.

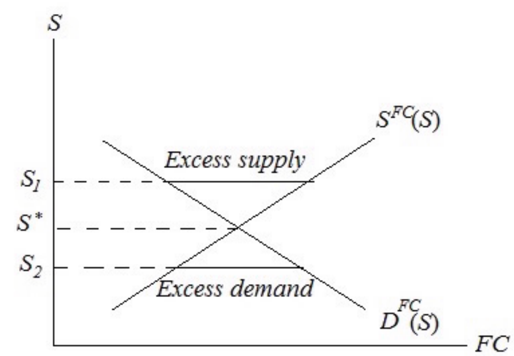

Figure 1.1 Exchange-rate equilibrium in the spot market

The spot exchange rate is determined in the spot market which is made up of financial institutions (e.g., commercial and investment banks, pension funds, hedge funds, money market funds, insurance companies, financial government entities) and non-financial institutions (e.g., corporations, non-financial government entities, private individuals) involved in buying and selling foreign currencies. According to BIS (2013) Triennial Survey, in 2013 the daily volume of spot contracts was $1.759 trillion (38% of total turnover). The majority of the spot trading was between financial institutions; only 19 percent of the daily spot transactions involved non-financial customers. This market is celebrated for the phrenetic rate at which it runs and the huge amount of money which is moved at a lighting speed in response to very small changes in price quotations.

Like in any other market, the demand for and supply of foreign exchange determine the price of a currency in the spot market, as shown in Figure 1.1. The figure describes a very simple (partial equilibrium) model of exchange-rate determination which serves as a useful introduction to the more general and complete models discussed in the next chapters.

In Figure 1.1 we display the amount of a foreign currency (FC) on the horizontal axis and the price, or exchange rate of that currency (S) on the vertical axis. Both supply and demand curves are plotted as well-behaved functions of the currency price, i.e., as increasing with the exchange rate for the supply function $S^{FC}(S)$, and decreasing with its price for the demand function $D^{FC}(S)$. In reality, there is no warranty that these functions are well-behaved. In particular, a real possibility that a currency supply curve slopes downward exits, rising the critical issue of explaining why foreign exchange markets are unstable. We shall examine this issue in Chapter , Section . Hence, in order to preserve simplicity we here overlook the conditions for exchange rate stability. For now you simply note that the odd case of a downward-sloping supply arise because on the horizontal axis we display *values* (e.g., the amount of euros, or dollars, or yen, etc.) and not *quantities* (e.g., number of cars, or phones, or wine bottles, etc.) as in traditional supply and demand setup. Values involve prices and quantities and they respond differently than do quantities (see).

The supply and demand curves of a foreign currency derive from the flow of payments between the residents of a country and the rest of world during a given time period. These flows are summarized in the balance-of-payments account, which records the country's international transactions with other nations (see Chap. 4). To understand, let us consider the supply curve first. It comes from the need by foreign residents to buy the domestic currency with foreign currency to pay for their purchases in the domestic country, i.e., to pay for domestic exports, holding of domestic assets, traveling to the domestic country. Being the exchange rate here defined as the price of domestic currency in terms of foreign currency (i.e., as the amount of domestic currency needed to purchase one unit of a foreign currency), a rise (depreciation) in S means that domestic exports, assets, travelling and tourism become cheaper for foreign residents. As such, they will start purchasing more domestic goods, assets and services, therefore leading to an increased demand of domestic currency which is purchased by increasing the supply of foreign currency. This yields an upward-sloping supply curve in the $[FC, S]$ plan.

To take a simple example, let us look at the euro-dollar exchange rate (€/\$), and imagine that the euro depreciates against the dollar moving from €0.903/\$1 to €0.982/\$1. Following the euro depreciation, the cost of EUR exports turns out to be cheaper for US resident and this will increase the demand for EUR exports and hence for euros which are purchased by increasing the amount of dollars supplied in the forex market. As a result, the supply curve for dollars happens to slope upward in the foreign exchange market.

Similarly, the demand curve for a foreign currency derives from domestic residents purchasing foreign goods and services, i.e., domestic imports, domestic investors purchasing foreign assets, and domestic tourists traveling abroad. In this case, a rise in S means that imports becomes costlier for domestic residents, leading to a reduced demand of foreign goods and with it to a reduced demand of foreign currency. This results in a downward-sloping demand curve in the $[FC, S]$ plan.

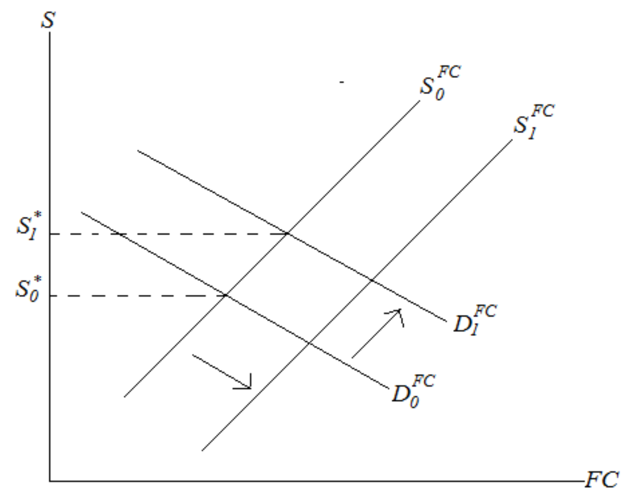

Figure 1.2 The effects of changes in demand and supply

In the example given above, a depreciation of the euro against the dollar makes the price of US exports to EUR importers to rise and this leads to a lower demand for US exports and hence dollars. Therefore, the demand curve for dollars slopes downward in the forex market.

Under a flexible exchange-rate regime, currency market equilibrium is found at a price such that the supply of and the demand for the foreign currency are equal. This is S^* in Figure 1.1. Exchange rates other than S^* cannot be sustained in the market, as they will tend to fall (appreciate) if there is an excess supply (S_1 in Fig.1.1) and to rise (depreciate) if there is an excess demand (S_2 in Fig.1.1).[1] It also follows from Figure 1.1 that any factor which results in a shift in the demand

[1]Under a fixed rate regime, on the other hand, the foreign exchange value of a currency is deliberately set by the government authorities, who actively act in the market to keep the fixed rate from changing. Alternative exchange-rate regimes are discussed in Chapter

for or supply of the foreign currency will result in price changes until a new equilibrium level is secured. These equilibrium price changes are shown in Figure 1.2.

2.3 The Forward Exchange Rate

The forward exchange rate is the rate that is agreed today for delivery of a currency at some future date. The rate is negotiated and settled at the time the contract is made but payment and delivery are not required until maturity. Banks typically quote forward rates for value dates of 1 month (30 days), 2 months (60 days), 3 months (90 days), 6 months (180 days), 9 months (270 days) and 12 months (360 days). However, actual contracts can be arranged for other lengths up to 5 or 10 years.

The forward exchange market is valuable for three main classes of activity, hedging, arbitrage and speculation. As stressed in Sect. 2.1, hedging is the activity of ensuring against the *exchange risk* coming from possible future fluctuations in the spot exchange rate. For example, an importer (exporter) who has to make (is to receive) a payment in foreign currency at a given future date can ensure himself against the exchange rate risk by buying (selling) the necessary amount of foreign currency in the forward market. Since the exchange rate is fixed now, he then knows exactly how much he will pay (receive) in domestic currency. Arbitrage and speculation are the activity of taking advantage of interest-rate and price differentials in the forward and spot market. For example, if a speculator believes a given foreign currency will appreciate in the future, he will buy that currency in the forward market. When the contract matures he will sell the currency on the spot market to make a profit (if he got it right). Similar considerations hold for arbitraging as we show below.

Forward contracts, in any case, are not the only way of carrying out these activities. Another possibility is to use the spot market. Therefore, the issue of which market is the best alternative arise naturally.

We shall look at this question by considering the case of an economic agent who need a foreign currency to make a payment at a future date, or to invest in foreign bonds, or to speculate.[2] For simplicity, let us focus on a one-period-forward market and denote the corresponding forward rate as $\mathcal{F}_{t,t+1}$, where t and $t+1$ are the day the forward contract

[2] The case of an agent who expects a payment at a future date is a mirror-image of this.

is negotiated and the day it is executed, respectively.[3] In addition, let $\mathcal{A}$, S_t, i_t and i_t^* be the amount of foreign currency due in the future, the date t spot exchange rate, and the one-period date t domestic and foreign interest rates, respectively. Under this conditions, the available alternative are as follows:

- the agent can buy the sum $\mathcal{A}$ on the forward market paying out next period $\mathcal{F}_{t,t+1}\mathcal{A}$ in domestic currency (*forward covering*);

- the agent can use the spot market to purchase an amount $\mathcal{A}/(1+i_t^*)$ of foreign currency, invest it in the foreign country at the interest rate i_t^* for one period to obtain $[\mathcal{A}/(1+i_t^*)](1+i_t^*) = \mathcal{A}$ at the maturity date (*spot covering*). The cost of $\mathcal{A}/(1+i_t^*)$ on the spot market is $S_t\mathcal{A}/(1+i_t^*)$; the opportunity cost (interest foregone on owned funds, or paid on borrowed funds) is $[S_t\mathcal{A}/(1+i_t^*)](1+i_t)$.

It then follows that the first alternative is better than, the same as, or worse than the second one according to whether

$$\mathcal{F}_{t,t+1}\mathcal{A} \lesseqqgtr S_t\mathcal{A}\frac{1+i_t}{1+i_t^*}, \tag{1.1}$$

that is, according to whether forward covering is cheaper, cost the same as, or is costlier than spot covering.

An alternative strategy called *currency swap* is to buy the foreign currency spot and sell it forward (*swap-in*), or sell the foreign currency spot and buy it forward (*swap-out*). More generally, a *swap* is an agreement to buy, or borrow and sell, or lending foreign exchange at a fixed exchange rate where the buying and selling are separated in time.[4]

Nevertheless, as long as the inequality in (1.1) persists, agents will choose forward (spot) covering until equality of both strategies holds, that is

$$\mathcal{F}_{t,t+1}\mathcal{A} = S_t\mathcal{A}\frac{1+i_t}{1+i_t^*}. \tag{1.2}$$

For example, if

$$\mathcal{F}_{t,t+1}\mathcal{A} < S_t\mathcal{A}\frac{1+i_t}{1+i_t^*},$$

agents will start buying the sum $\mathcal{A}$ on the forward market driving up the forward price $\mathcal{F}_{t,t+1}$, if interest rates do not change, until forward covering were no longer below spot covering. Similarly, if

$$\mathcal{F}_{t,t+1}\mathcal{A} > S_t\mathcal{A}\frac{1+i_t}{1+i_t^*},$$

[3] The period length is arbitrary and may also be read as 1 month, 2 months or any other relevant interval with no loss of generality.

[4] See, e.g., Levi (1996), pp. 66-67.

agents will sell $\mathcal{A}$ on the forward market, driving down the forward price until indifference of both strategies holds.

Condition (1.2) is called the *neutrality condition* and the forward rate is said to be at the *parity*, or at *interest parity* when it prevails. To see, in (1.2) divide through by $S_t\mathcal{A}$ to obtain

$$\frac{\mathcal{F}_{t,t+1}}{S_t}=\frac{1+i_t}{1+i_t^*}, \tag{1.3}$$

whence, by subtracting one from both sides,

$$\frac{\mathcal{F}_{t,t+1}-S_t}{S_t}=\frac{i_t-i_t^*}{1+i_t^*}. \tag{1.4}$$

The left-hand side of (1.4) shows the divergence between the forward exchange rate and the corresponding spot rate called *forward margin*. When the foreign currency is more expensive for forward delivery than for spot delivery, the foreign currency is said to be at a *forward premium*; when the foreign currency costs less for forward delivery than for spot delivery, the currency is said to be at a *forward discount*. When the forward and spot rate are equal, the foreign currency is said to be *flat*. Accordingly, the forward margin gives a measure of the forward premium (discount) of the currency as a percentage of the spot rate by multiplying by 100.[5]

The right-hand side of (1.4) is the interest rate differential between the domestic and foreign country if we ignore the term $(1+i_t^*)$.[6] Equation (1.4) thus states that the forward premium/discount of a foreign currency should be equal to the interest rate differential between the domestic and foreign currency/country in order to prevent arbitrage. Condition (1.4) is also known as *covered interest parity* (CIP), since there is no advantage to covered borrowing, or investing in any particular currency, or from covered interest arbitrage when it holds. The mechanism leading to (1.4) is illustrated in the following example.

Let the period length be fixed over a 3-month horizon and consider an investor who has € 1,000,000 to invest for 3 months. Let the spot euro-dollar rate be €0.91022/$1 and the annual interest rate on

[5]More generally, if we denote by $\mathcal{F}_{t,t+n}$ the n-period forward rate and allow for compound interest assuming constant interest rate, the parity condition can be written as

$$\frac{\mathcal{F}_{t,t+n}}{S_t}=\left(\frac{1+i_t}{1+i_t^*}\right)^n,$$

or

$$\left(\frac{\mathcal{F}_{t,t+n}}{S_t}\right)^{\frac{1}{n}}=\frac{1+i_t}{1+i_t^*}.$$

[6]This is legitimate only for low value of i_t^*

EUR and USD be 0.7% and 0.4%, respectively. Using (1.3), the 3-month forward exchange rate of euro to dollar that leaves no arbitrage opportunity is[7]

$$0.91090 = 0,91022 \times \left[\frac{1 + 0.007 \times (90/360)}{1 + 0.004 \times (90/360)}\right].$$

Under this rate our investor would be indifferent between investing in domestic-currency denominated securities and earn a 3-month return of 0.175%, which yields €$1,001,750$=€$1,000,000 \times (1+0.00175)$, or investing in foreign-currency denominated securities purchased at the spot rate of 0.91022 and earn a 3-month return of 0.1%, which yields $\$1,099,734 = 1,000,000 \times (1/0.91022) \times (1 + 0.001)$, and re-converting to domestic currency at the free-arbitrage forward rate of 0.91090 to finish up with €$1,001,750$ =$\$1,099,734 \times 0.91090$. Any advantage he might get from the higher domestic rate of interest would be offset exactly by the poorer exchange rate when he converts his USDs to EURs. This result checks with (1.4) which shows that the forward margin, or the dollar forward premium (equal to $0.075\% = [(0.91090 - 0.91022)/0.91022] \times 100$) offsets exactly the interest rate differential between the two currencies/countries (equal to $0.075\% = 0.175\% - 0.1\%$).

If this were not to be true, arbitrage opportunity exists and investors will start a process of moving from one currency to another in order to take advantage of the opportunity for profit. For example, if the 3-month forward rate were €0.91119/$1, the forward margin ($0.107\% = [(0.91119 - 0.91022)/0.91022] \times 100$) would be greater than the interest rate differential $(0,075\%)$. So it would pay for our investor to buy dollars at the spot rate of 0.91022, invest in US securities for 30 days and simultaneously sell dollars at the 3-months forward rate of 0.9119 to cover for the exchange risk. At the end of period, using the forward contract he converts the dollars back into euros to finish up with €$1,002,067 = 1,099,734 \times 0.91119$, thus earning an additional profit over investing in euros of €$317 = 1,002,067 - 1,001,750$. However, as long as the above disparity persists a large numbers of investors would see the benefit of this strategy and would follow suit. In other words, they would buy dollars spot (forcing up the spot rate above 0.91022); buy US bonds (forcing their price up and their return down); and sell dollars on the forward market to cover for the exchange risk (forcing down the 3-month forward rate below 0.91119). This process

[7]Notice, that the multiplication of the annual (home and foreign) interest rate by $0.25 = (90/360) = (1/4)$ is required to find the 3-month return. Similarly, if the forward horizon were restricted to be 1 month, 6 months, or 9 months, the multiplication factor would be $(30/360) = (1/12)$, $(180/360) = (1/2)$, $(270/360) = (3/4)$, respectively.

will continue until the neutrality condition (1.4) prevails. Of course, an opposite strategy and process of adjustment follows if the above inequality between the forward margin and the interest rate differential is reversed.

It should be point out that condition (1.4) does not hold exactly in the real world. The possible explanations include: transaction costs, capital controls, political and credit risks, difference in tax rates on interest income and foreign exchange losses/gains, liquidity differences between domestic and foreign securities. However, since deviations from (1.4) appear to be small, at least for all major traded countries, we probably commit no great error from a macroeconomic point of view by assuming that the forward parity holds exactly when restrictions on capital movements are absent.[8]

Equation (1.4) often appears in the literature in a simplified form, which we use in the next chapters to show the connection with two other important parallel conditions that apply to financial markets: the purchasing-power parity principle (PPP) and the uncovered interest parity condition (UIP). Let $f_p = (\mathcal{F}_{t,t+1} - S_t)/S_t$ denote the forward premium/discount. Then $(\mathcal{F}_{t,t+1}/S_t) = 1 + f_p$, and we can rewrite condition (1.3) as

$$1 + f_p = \frac{1 + i_t}{1 + i_t^*}.$$

If we now take logs of both members of this equation, recalling that $\ln(1 + x) \simeq x$, we get

$$f_p = i_t - i_t^*,$$

whence

$$i_t = i_t^* + f_p. \tag{1.5}$$

Equation (1.5) says that international investors should be indifferent between domestic- and foreign-currency denominated securities if the domestic-currency interest rate equals the foreign-currency rate plus the forward margin. When the domestic-currency interest rate is less than the sum of the foreign-currency rate plus the forward exchange premium/discount, international agents should invest in the foreign currency; when the domestic-currency rate exceeds the sum $(i_t^* + f_p)$, agents should invest at home. Only when (1.5) holds investors have no incentive to move their funds from where they are placed.

[8]See, e.g., Baille and McMahom (1989), Frankel (1993, ch. 2), De Vries (1994), Levi (1996, pp. 271-289).

2.4 Nominal, Real and Effective Exchange Rates

The nominal exchange rate

In Section 1.2 we defined the exchange rate as a price, specifically the relative price of two currency which simply states the rate at which one currency can be exchanged for another currency. It is 'nominal' because it measures only the value of one country's currency in terms of another with no reference to other aspects such as the purchasing power of that currency. The symbols S_t and $\mathcal{F}_{t,t+1}$ ($\mathcal{F}_{t,t+n}$) we introduced in the previous sections thus denote the nominal exchange rates in the spot and forward market, respectively.

In a fixed rate regime, the nominal exchange rate is determined a by national authorities (central banks); in a flexible rate regime the nominal rate is determined in the forex market by demand and supply for the two currencies. Rates are usually settled in continuos quotation, with newspapers reporting daily quotations as average or end-of-day quotation in a specific financial market.

Recall that in this book we chose the price quotation system to express nominal quotes. Accordingly, if in a newspaper, for example, we read the end-of-day quotes shown in Table 1.1, we understand they denote the number of euros (€), pounds (£), yen (¥), Canadian dollars (C$), Australian dollars, and yuan (CNY) that are required to buy one US dollar ($). An increase in these rates is called *nominal depreciation* of the (home) currency (e.g., a change in the €/$ rate from 0.88067 to 0.99225 is 12.67% nominal depreciation of the euro against the dollar); a decrease is termed *nominal appreciation* (e.g., a change in the £/$ rate from 0.63686 to 0.62580 is a 1.74% nominal appreciation of the pound against the dollar).[9] Changes in the nominal value of a currency, however, do not necessarily imply that the country's competitiveness has changed. For such a measure we have to look at the real exchange rate.

Table 1.1 Hypothetical nominal exchange rates

	€	£	¥	C$	A$	CNY
$	0.88067	0.63686	138.85	1.31168	1.36351	6.38843

[9] Under a fixed exchange-rate regime, a downward adjustment of the rate by the central bank is termed revaluation, and an upward adjustment devaluation.

The real exchange rate

The real exchange rate, like the nominal one, is a relative price obtained from the nominal exchange rate adjusted for the relative price levels between the home and the foreign country. It gives a measure of competitiveness, which tells how much the goods and services in the domestic country can be exchanged for the goods and services in a foreign country. Formally, letting P, P^* and S denote the price level in the home country, the price level in the foreign country, and the (spot) nominal exchange rate, respectively, we can write the date t real exchange rate, Q_t, as

$$Q_t = \frac{S_t P_t^*}{P_t}. \tag{1.6}$$

A rise in Q is called *real depreciation*, and can be the result of a nominal depreciation (a rise in S), a rise in the relative prices (P_t^*/P_t), or both. It signals a real depreciation of the home currency, which means that foreign goods are now more expensive for domestic residents (the domestic price of a bundle of foreign goods has risen). For example, if in a given period the yen/dollar nominal rate changes from $120,70$ to $132,77$ (a 10% nominal depreciation) while the relative prices remain unchanged, say to $(P_t^*/P_t) = 100/100 = 1$, then the ¥/\$ real rate $\left(Q^{¥/\$}\right)$ undergoes a real depreciation of 10%, thus signaling that a basket of US goods costs now relatively more for JP residents as they have to pay ¥$132,77$ to purchase the original bundle of US goods of ¥$120,70$. The same result obtains if we let the nominal rate be unchanged and the US price index rise of 10% (from 100 to 110), or take any convex combination of changes in nominal rate and relative prices that ends up in an upward rise of 10% (e.g., a 5% increase in both S_t and P_t^*/P_t).

A fall in Q is termed a *real appreciation*, and can be the result of a nominal appreciation, a decrease in the relative prices, or both. It signals a real appreciation of the home currency, and means that foreign goods are now less expensive for domestic residents. A simple example follows by simply reversing the changes applied in the above real depreciation case.

It should now be point out that in the literature there are a number of alternative ways of measuring the relative prices and hence the real exchange rates. A useful though not exhaustive list is given below.[10]

- If all goods are postulated to be traded internationally, $P = P^X$, that is, the domestic price index equals the price of exports $\left(P^X\right)$, and similarly $P^* = P^{\mathfrak{M}}$, where $P^{\mathfrak{M}}$ is the price of imports.

[10]More detailed and excellent surveys can be found, e.g., in Maciejewski (1983), Edwards (1989b), Hinkle and Montiel (1999), Harberger (2001, 2004), Chin (2006).

Substitution into (1.6) yields

$$Q_t = \frac{S_t P_t^{\mathfrak{M}}}{P_t^X}, \tag{1.6a}$$

which relates the real exchange rate to the inverse of *terms of trade* (the ratio between the exports and imports price $P^X/P^{\mathfrak{M}}$).[11] Equation (1.6a) tells the amount of exports that are required to obtain one unit of import. As a result, an increase in Q signals a deterioration in the terms of trade, as it means that a grater amount of exports are required to obtain one unit of imports, whereas a fall denotes an improvement in $P^X/P^{\mathfrak{M}}$.

- If the price index is supposed to be a geometric average of so called traded- and nontraded-goods (i.e., goods that are traded or not internationally), both the domestic and foreign price level can be written as

$$\begin{aligned} P_t &= \left(P_t^N\right)^{\gamma} \left(P_t^T\right)^{1-\gamma} \\ P_t^* &= \left(P_t^{*N}\right)^{\delta} \left(P_t^{*T}\right)^{1-\delta}, \end{aligned}$$

where the superscripts N and T stand for non-tradable and tradable goods, and γ and δ are the shares of non-tradables in the general price index for the domestic and foreign economy, respectively.[12] Substituting into (1.6) yields

$$Q_t = \left(\frac{S_t P_t^{*^T}}{P_t^T}\right) \left[\frac{\left(P_t^{*^N}/P_t^{*T}\right)^{\delta}}{\left(P_t^N/P_t^T\right)^{\gamma}}\right],$$

or, letting for simplicity $\delta = 0$, i.e., assuming that the foreign economy produce only tradable goods (so that $SP^{*T} = P^T$, if PPP holds; (see, Chapter, Sect.),

$$Q_t = \left(\frac{P_t^T}{P_t^N}\right)^{\gamma}. \tag{1.6b}$$

[11] Things do not change, however, if we recognize that exports and imports are parts of domestic and foreign output.

[12] Generally, the key element to consider when looking at the tradable and non-tradable classification is where the price for the good (or service) is determined. If it takes place in the world market, the good should be considered tradable. If the price determination takes place in the local market, the good should be considered non-tradable. Water supply, all public services, hotel accommodation, real estate, construction, local transportation are examples of nontradable goods; cars, electronics, clothing, machinery and equipment, oil and raw materials are example of traded goods.

> Equation (1.6b) defines Q as the internal relative price incentive for producing (or consuming) tradable goods as opposed to non-tradable goods, which makes it an indicator of incentives to allocate resources in the home country. As a consequence, an decrease (increase) in Q indicates that in the home country incentives to allocate resources in the less competitive, non-tradable sector have risen (declined).

If cost competitiveness is a focus of attention, another alternative is the relative unit labour costs between the domestic and foreign economy. In this case, the real exchange rate is defined as

$$Q_t = \frac{S_t \mathcal{W}_t^*}{\mathcal{W}_t}, \tag{1.6c}$$

where $\mathcal{W}$ and $\mathcal{W}^*$ are unit labour costs, respectively.[13] An rise (decrease) in foreign relative to domestic unit labour costs $(\mathcal{W}_t^*/\mathcal{W}_t)$ means an improvement (deterioration) in the external competitiveness of the home country, which causes the real exchange rate to appreciate (depreciate) and the real value of the currency to depreciate (appreciate). This is because a rise in $(\mathcal{W}_t^*/\mathcal{W}_t)$ is reflected in a rise of relative price (P^*/P) and hence of Q as shown in Equation (1.6).

The effective exchange rate

So far, the discussion of the (nominal and real) exchange rate has involved only two countries (currencies). This is a convenient simplification in that it allows one to define the theoretical construct while abstracting from third-country effects. Nevertheless, in the real world a country typically trade not with a single country but with a number of countries, so it is crucial to have a measure of the overall external value of a country's currency relative to all other currencies being traded. This is what the effective exchange rate aims to. It is usually presented as an index number with a base of 100, so as to show an appreciation when it increases (index above 100) and a depreciation when it decreases (index below 100). Effective exchange rates index are computed and published by international financial institutions such as the International Monetary Fund (IMF), the World Bank (WB), Bank of International Settlements (BIS), by central banks and private institutions.

Like the two-country case, the are two different measures: the *nominal effective exchange rate* and the *real effective exchange rate*. The nominal effective exchange rate (NEER) measures the average value of

[13] Unit labor costs are routinely computed as ratio between nominal wage rate and labour productivity.

a country's exchange rate against all other trading partners. Formally, it is computed as

$$S_t^{i,NEER} = \left(\frac{1}{S_t^{1i}}\right)^{\omega_1} \times \left(\frac{1}{S_t^{2i}}\right)^{\omega_2} \times \cdots \times \left(\frac{1}{S_t^{ni}}\right)^{\omega_n},$$

or

$$S_t^{i,NEER} = \prod_{j=1,j\neq i}^{n} \left(\frac{1}{S_t^{ji}}\right)^{\omega_j}, \sum_{j=1,j\neq i}^{n} \omega_j = 1, \tag{1.7}$$

where $S_t^{i,NEER}$ is the nominal effective exchange rate of currency/country i, S_t^{ji} the nominal exchange rate of currency/country i relative to currency/country j, expressed in direct quotation form (the price of foreign currency in terms of domestic currency), ω_j the weight assigned to currency j in the computation of the index, usually based on bilateral trade volumes (the sum of exports and imports, expressed as a proportion of total exports and imports), and $\prod$ a symbol denoting the product of elements $\left(1/S_t^{ji}\right)^{\omega_j}$. Equation (1.7) shows that the NEER is computed as a geometrically weighted average (or convex combination) of nominal bilateral exchange rates, where the sum of weights is equal to one by definition. Observe, that in order to make changes in the $S^{i,NEER}$ index consistent with changes (appreciation or depreciation) in the nominal bilateral exchange rates S^{ji} we used the inverse of direct quotation $(1/S^{ji})$ and not simply the direct quotation (S^{ji}), as it would have been the case had we used the volume quotation system.[14]

The real effective exchange rate (REER) is an overall measures of a country's competitiveness against the other trading partners. It can be computed as in (1.7) using real rather than nominal bilateral exchange rates. Formally, we have

$$Q_t^{i,REER} = \prod_{j=1,j\neq i}^{n} \left(\frac{1}{Q_t^{ji}}\right)^{\omega_j}, \sum_{j=1,j\neq i}^{n} \omega_j = 1, \tag{1.8}$$

which computes the real effective exchange rate of country i $\left(Q^{i,REER}\right)$ as a geometrically weighted average of real bilateral exchange rates Q^{ji}.

[14] A linearized version of (1.7), often found in the literature, is

$$\begin{aligned} s_t^{i,NEER} &= -\left(\omega_1 s_t^{1i} + \cdots + \omega_n s_t^{ni}\right) \\ &= -\sum_{j=1,j\neq i}^{n} \omega_j s_t^{ji}. \end{aligned}$$

It can be obtained from (1.7) by taking logs of both sides, and denoting log variables with lower-case letter.

Chapter 3

Currency Derivatives and Options Markets

3.1 Introduction

We learned from previous Sections that the forex market is a very large market with many different features, advantages and risks. We also learned that forex investors may trade in futures as well as spot markets. Yet, when looking at the forward markets we hear of a number of financial instruments that can be used by agents for speculation, arbitrage and hedging activities. These financial instruments are known as currency derivatives and they have seen an enormous growth since 1970. This Section gives an overview to the main types of derivative instruments used in the forex market: futures, options and swaps.

Financial derivatives are traded either on organized derivatives exchanges (ODE) markets, such as the Chicago Board Options Exchange (CBOE), the Japan Exchange Group (JPX), the New York Stock Exchange London international financial futures exchange (NYSE Liffe), etc., or in Over-The-Counter (OTC) markets. The differences between these two competing market segments are not only from where the trading takes place but also how. In the exchange-traded segment, derivatives contracts are highly standardized with specific delivery or settlement terms. Trades and prices are publicly reported and cleared in a clearing house, making risks and market trends transparent to all market participants and regulators. In addition, the clearing house is obliged to honour the trade if the seller defaults and the solvency of the clearing house is protected by marking all positions to market daily through a system of margins.

By contrast, in the OTC segment derivatives contracts are traded bilaterally and arranged on a tailor-made basis. All contract terms regarding the underling asset, contract size, price, maturity and other features are negotiable between the two parties. Transactions are settled by telephone or other communication means, and prices are not reported publicly.

These different features of ODE and OTC derivatives markets mean that they can both complement and compete with each other. For example, OTC derivatives can rely on high liquid and price transparent ODE markets to dynamically hedge their market risk. Conversely, ODE derivatives can face competitive pressure from more flexible OTC markets on price and services.[1]

3.2 Futures

A currency future, or FX future is a futures contract to exchange one currency for another at a given date in the future at a price (exchange rate) that is fixed on the purchase date.

3.3 Options

A currency option, or FX option is a contract that grants the holder the right, but not the obligation, to buy or sell a currency at a pre-agreed price (exchange rate) for a specified time period.

3.4 Swaps

A currency swap, or FX swap is a simultaneous purchase and sale of identical amounts of one currency for another with two different value dates (normally spot to forward).

To be completed.

[1] See, e.g., Nystedt (2004), Switzera and Fanb (2008), Deutsche Börse Group (2008), Prabha, Savard and Wickramarachi (2014).

Part II

The Balance of Payments Accounts

Chapter 4

Balance-of-Payments and International Investment Position

4.1 Introduction

In this chapter we shall look at the balance-of-payments (BP) account and International Investment Position (IIP) which are among the major economic indicators for policymakers and private agents in an open economy. The main reason for such a prominent role is that they gives an official account of all transactions and positions between an economy and the rest of world, so signaling the performance of a country in international markets including trading and capital flows with other nations, exchange rate policy, reserves management, and external vulnerability.

The major purpose of this chapter is to provide a comprehensive summary of what data are included in these accounts, how they are compiled and classified and of possible economic interpretations of the statistics. We discuss the general accounting principles guiding the compilation of the basic structure of a BP account and of different components that are included. We also discuss the meaning of surplus, deficit and equilibrium in the main accounts, and how the BP and IIP accounts are related. We finally look at how balance of payments accounts and data on the international investment position can be interpreted within the framework of the national income accounts, so fitting the main macroeconomic variables and accounting relations in an open economy context.

4.2 System Structure and Accounting Principles

The balance of payments is a statistical statement that systematically summarizes all the economic and financial transactions of a country with the rest of the world over a specific time period. These transactions involve interactions between residents and nonresidents, or foreign residents of a country, and include payments for the country's exports and imports of goods, services, financial capital, and financial transfers.[1] The international investment position, on the other hand, is a statistical statement that shows the value and composition of the stock of an economy's financial assets and liabilities with the rest of the world. Therefore, the balance of payments records the flows of payments in a given time period, whereas the international investment position is concerned with financial stocks at given point in time. Figures on these accounts are normally reported in the domestic currency of the compiling country and calculated over a month, a quarter or more commonly over a year.

4.2.1 Accounting Principles

The accounting principles behind balance-of-payments statistics derive from the double-entry bookkeeping system, which means that every

[1] The notions of 'resident' and 'nonresident' are however problematic, as citizenship and residency do not necessarily coincide from the viewpoint of BP statistics. The International Monetary Fund (IMF) in its *Manual* provides a set of rules to solve doubtful cases. The general criterion for determining residence builds around the concept of center of predominant economic interest defined as "*the economic territory to which each entity is most closely connected*" (see, IMF, 2009, p. 89). For example, all members of the same household have the same residence as the household itself, even though they may cross borders to work or otherwise spend periods of time abroad; similarly, corporations and other institutions have the residence in the economy in which they are legally constituted and registered. By contrast, international organization such as the International Monetary Fund, the World Bank, the United Nations, etc. are regarded as foreign resident even though they may be located in the compiling country.

recorded transaction is represented by two entries with equal values but opposite signs, a debit (-) and a credit (+). More specifically, under the conventions of the system a compiling economy records:

- credit entries for all transactions involving exports of goods and services, income receivable, and reduction in assets or increase in liabilities;
- debt entries for all transactions involving imports of goods and services, income payable, and increase in assets or reduction in liabilities.

Thus, a schematic representation is as follows:

Exports of goods and services	Credit	(+)
Imports of goods and services	Debt	(–)
Increase in liabilities	Credit	(+)
Increase in assets	Debt	(–)
Decrease in assets	Credit	(+)
Decrease in liabilities	Debt	(–).

A key point about the double-entry bookkeeping is that in an accounting sense the balance of payments is always in balance. This is because the sum of all credits should be equal to the sum of all debits, and the overall total should equal zero. To understand how this works let us take a couple of simple examples.

Suppose that Italy exports €10 billion worth of goods to the United States and that the US importers pay from euro accounts that are kept in Italian banks. Under the double-entry system, the value of exports will be represented by a credit entry in the Italy's balance of payments and the financial asset acquired (bank draft or any other credit instrument) by an offset debit entry. Suppose, on the other hand, that Italian residents buy €5 billion worth of US securities and that the US sellers put the €5 billion they receive into Italian bank accounts. In this case, the increase in foreign assets is recorded as a debt entry and the payment as an offsetting credit entry. These BP recording rules are set out in Table 1.2.

Table 1.2 Examples of Balance of Payments Recording

	Credit	Debt
Exports of goods	10	
Bank deposits (increase in financial assets)		-10
Increase in foreign assets		-5
Bank deposits (reduction in financial assets)	5	

Nonetheless, the accounts never add to zero in practice, either because data estimates are often derived from different sources and may be incomplete, inconsistent and subject to inevitable measurement errors or because timing and valuation effects along with a variety of other factors tend to cause imbalances in the information recorded. As a consequence, a separate entry, equal to the amount of the resulting imbalances with the sign reversed and labelled *net errors and omissions,* is included to balance the overall account (see next Section).

From a foreign exchange perspective, however, a key point to emerge from the above examples and, more generally, from a county's balance-of-payments statistics is that any transaction that is recorded as a credit entry represents a demand for its home currency (or a supply of foreign currency) in the foreign exchange market. Conversely, any transaction that is recorded as a debt entry represents a supply of its home currency (or a demand for foreign currency) in the foreign market. Therefore, credit entries imply demands for a country's home currency and result from exports of goods and services, income receivable, and sell of financial and real assets to foreign residents. Similarly, debt entries imply supplies of a country's home currency and result from imports of goods and services, income payable, and purchase of financial and real estate to foreigners. Since the BP statistics record all the economic transactions of an economy with the rest of the world, it follows that they also include a list of all potential factors that lie behind the demand and supply curve for a currency. This is what makes the balance-of payments account such a useful and powerful framework for understanding the factors that influence the supply and demand of a currency and hence the exchange rate.

The full meaning of this statement will become clear as we consider the basic structure and classification system of BP and IIP statistics.

4.2.2 Standard Components

According to the Sixth edition (2009) of the IMF's Balance of Payments and International Investment Position Manual (BPM6), the standard components in the BP framework are comprised of two main sections: the current account and the capital and financial account. The current account includes all transactions that pertain to goods and services, income, and current transfers. The capital and financial account pertains to capital transfers, transactions in nonproduced nonfinancial assets, and financial assets and liabilities. Examples of balance-of-payments for Italy and the euro area are in Table 1.3.

Table 1.3 Summary Balance of Payments, 1st Quarter, 2015 (EUR billions)

	Italy			Euro area (EU19)†		
	Credits	Debits	Net	Credits	Debts	Net
1) Current Account	131.9	131.0	0.9	833.3	778.8	54.5
Goods	96.0	86.6	9.4	496.1	428.9	67.2
Services	18.2	20.5	-2,4	166.9	157.1	9.8
Primary incomes	14.7	13,8	1.0	146.9	118.9	28.0
Secondary incomes	3.0	10.1	-7.1	23.4	73.9	-50.5
2) Capital Account	0.6	0.9	-0.3	8.4	4.5	3.9
3) Net Lending/Borrowing to/from ROW‡ (1+2)			0.6			58.4
4) Financial Account	4.8	11.1	-6.3	504.8	513.7	-8.8
Direct investment	8.6	3.8	4.8	159.7	74.7	85.0
Portfolio investment	59.9	67.8	-7.9	129.0	260.2	-131.2
Financial derivatives			-1.7			27.3
Other investment	3.4	4.5	-1,1	182.8	178.7	4.1
Reserve assets			-0.4			6.0
Errors and omissions			-6.9			-67.2

†EU 19: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Portugal, Slovakia, Slovenia, and Spain.
‡ROW: Rest of the World.
Source: BOI (2015), ECB (2015).

Current Account

The current account includes all transactions that pertain to goods, services, and primary and secondary incomes (or income and current transfers).

- Goods:

comprises *general merchandise* (goods that residents export to, or import from, nonresidents), *goods for processing* (exports or imports of goods crossing the frontier for processing abroad and subsequent re-import or export of the goods), *repairs on goods* (repair activity on goods provided to or received from nonresidents on ships, aircraft, etc.), *goods procured in ports by carriers* (goods such as fuels, provisions, stores, and supplies that resident/nonresident carriers (air, shipping, etc.) procure abroad or in the compiling economy), and *nonmonetary* *gold* (exports and imports of all gold not held as reserve assets by the authorities).

Goods are recorded according to the fob (free on board) definition, both for exports and for imports, so that they are valued at the frontier of the exporting country. The receipts for exports are recorded as credits, the payments for imports are recorded as debits, and the difference (Net in Tab.1.3) is referred as *goods balance.* When the balance is in surplus this means that the revenues for exports are greater than the outlays for imports (credits>debits) and the difference appears with a positive or no sign in the balance of payments. Conversely, when the balance is in deficits (credits<debits) the difference appear with a negative sign.

- Services:

comprises *transportation* (services such as freight, passenger transportation by all modes of transportation and other distributive and auxiliary services that are performed by residents for nonresidents and vice versa), *travel* (goods and services acquired by nonresident travelers for business and personal purposes during their visits of less than one year in a country), *other services* (service transactions with nonresidents not covered under transportation or travel, such as communication services, construction services, insurance or financial services).

As for goods, credits denote export revenues, debits import payments, and the difference (Net) the surplus (+) or deficit (-) in the balance of service. The sum of balance on goods and services is often referred as *trade balance.*

- Primary income:

shows income flows between residents and non residents in return for providing temporary use to another entity of labor, financial resources, or nonproduced nonfinancial assets. It includes *compensation of employees* (wages, salaries and other benefits, including social contributions and private insurance policies or pension funds), *investment income* (dividends, withdrawals from income of quasi-corporations, reinvested earnings, interest, and investment income attributable to policyholders in insurance, standardized guarantees and pension funds), and *other primary income* (rents, taxes and subsidies on products and production).

- Secondary income:

shows current transfers that are not transfers of capital between residents and nonresidents without anything of economic value being supplied as a direct return. It comprises *government transfers* (current taxes on income and wealth, social contributions and social benefits, current international cooperation, miscellaneous current transfers, etc.) and *transfers of other sectors* (workers' remittances, insurance premiums, claims on non-life insurance, and other transfers such as fines and penalties, gifts and donations, etc.).

As for goods and services, inflows from nonresidents are considered as credit entries, outflows as debit entries, and the difference as the surplus or deficit in the income balance.

Computation of the subtotal up to the secondary income account yields the *current account* of the balance of payments, which shows the amount of credits and of debits in the goods, services and income accounts (see, Tab 1.3). When the sum of exports and income receivable exceeds the sum of imports and income payable the current account balance (Net) is in surplus; conversely, when total imports and income payable exceeds exports and income receivable it is in deficit. Table 1.3 shows that in 1st Quarter of 2015 the current account balances for Italy and the euro area were in surplus by € 0.9 billion and by € 54.5 billion, respectively.

Capital Account

The capital account shows credit and debit entries for nonproduced nonfinancial assets and capital transfers between residents and nonresidents.

- Nonproduced, nonfinancial assets:

refers to transfers of ownership between residents and nonresidents of *natural resources* (land, mineral rights, forestry rights, water, fishing rights, air space, and electromagnetic spectrum); *licenses, leasing contracts and other contracts* (intangibles such as marketable operating leases, permissions to use natural resources not recorded as outright ownership of those resources or to undertake certain activities including some government permits, and entitlements to purchase a good or service on an exclusive basis); and marketing assets (brand names, trademarks, logos, etc.) and goodwill.

- Capital transfers:

refers to transfers of *ownership of fixed assets*; transfers of *funds linked to the acquisition or disposal of fixed assets* and the *forgiveness of debts.* Capital transfers are classified into two sectorial components: general government (capital taxes, debt forgiveness, investment grants and the other capital transfers) and other sectors (migrants' transfers, debt forgiveness and other transfers).

Net Lending/Borrowing

The sum of the balances on the current and capital accounts shows the net lending (surplus) or net borrowing (deficit) by the compiling country with the rest of the world. The reason the balance item net lending/borrowing is reported is that it reflects the amount of financial assets that are available for lending or needed for borrowing to finance all transactions with non residents. It is conceptually equal to the net balance of the financial account, although in practice the equivalence is hardly got.

Financial Account

The financial account records all transactions in external financial assets and liabilities. It is structured around five accounts, differentiated by the type of financial assets/liabilities involved in the transaction.

- Direct investment:

involves cross-border investments associated with residents in one country having control or significant influence over the management of firms resident in another country, and tends to be associated with lasting relationships. Direct or indirect possession of 10 per cent or more of the voting rights is proof of such a relationship. Direct investment is classified according to the instrument involved and comprises shares, other equity, reinvested earnings and debt instruments.

- Portfolio investment:

covers transactions between residents and non-residents involving debt and equity securities not included under direct investment. Portfolio investment is classified according to shares, investment fund shares, debt securities (short or long-term) and divided by resident sector and counterparty sector.

- Financial derivatives:

covers financial instruments linked to other financial instruments through which specific financial risks can be traded in financial markets. Transactions and position recorded under this item are those in options, futures, swaps, forward foreign exchange contracts and credit derivatives.

- Other investment:

involves positions and transactions other than those included in direct investment, portfolio investment, financial derivatives and reserve assets. It comprises: equity other than securities; currency and deposits; loans, insurance, pension schemes and standardized guarantees; trade credit and advances; other accounts receivable/payable; and SDR allocations (SDR holdings are included in reserve assets).

- Official reserves (reserve assets):

comprises external assets that are readily available to and controlled by monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets to affect the currency exchange rate, and for other related purposes (such as maintaining confidence in the currency and the economy, and serving as a basis for foreign borrowing). They include monetary gold, special drawing rights (SDR) holdings, reserve position in the IMF, foreign currency and deposits, securities (including debt and equity securities), financial derivatives, and other claims (loans and other financial instruments)

Errors and Omissions

This item is the result of errors and omissions in the compilation of balance-of-payments statements. It is derived residually from the net balance of the financial account minus the same item derived from the sum of the current and capital accounts, or net lending/net borrowing. Table 1.3, for example, shows that net lending/net borrowing measured from the current and capital accounts for Italy is 0.6, while the net balance measured from the financial account is -6.3; then, errors and omissions is -6.9.

Because the net lending or net borrowing derived from the current and capital account should in principle be equal to the overall balance on the financial account - for a surplus of credits over debits in the current and capital accounts there is a balancing net acquisition of financial assets or reduction of liabilities which is shown in the financial account -, it follows that a positive value of (net) errors and omissions signals that credit entries have been understated or debit entries have been overstated, and vice versa in case of a negative value.

International Investment Position

Closely related to the flow-oriented balance of payments framework is the stock-oriented international investment position. It is a statistical statement that shows at a point in time the value and composition of (i) *financial assets* of residents of an economy that are claims on nonresidents and gold bullion held as reserve assets, and (ii) *liabilities* of residents of an economy to nonresidents. The difference between the two sides of the balance sheet measures the economy's net IIP, which may be positive or negative. As shown in Table 1.4, the IIP presentation format comprises the same items as the financial account, namely, direct investment, portfolio investment, financial derivatives, other investment, and official reserves. Table 1.4 signals a net IIP for Italy and the euro area at the end of 2015-Q1 of € -478.3 and -1,294.5 billions, respectively.

The balance of payments and international investment position can be reconciled, as the change in the stock of external financial assets and liabilities in a period is attributable to financial flows (transactions on the financial account of the balance of payments) and valuation adjustments (referring to the changes between the start and the end of the period in exchange rates and the prices of underlying assets and any other adjustments). The next section discusses in more detail the issue.

Table 1.4 International Investment Position, 1st Quarter, 2015
(end-of-period stocks in billions of euros)

	Italy			Euro area (EU19)		
	Assets	Liabilities	Net	Assets	Liabilities	Net
Direct investment	564.9	418.1	146.8	8,204.4	6,331.7	1,872.7
Portfolio investment	1,073.2	1,481.4	-408.2	7,270.9	10,995.3	-3,724.4
Financial derivatives	131.3	200.7	-69.4	-21.0		-21.0
Other investment	466.6	744.1	-277.5	5,029.9	5,054.9	-25.0
Reserve assets	130.0		130.0	603.1		603.1
Total net position	2,366.0	2844.3	-478.3	21,087.3	22,381.9	-1294.6

Source: BOI (2015), ECB (2015).

Chapter 5

Some Linkages between BP, IIP, and National Accounts

5.1 Introduction

This section discusses the major links among the balance of payments accounts, international investment position and the broad system of national accounts. The goal is to: (i) draw from the flows, stocks, and other changes affecting the level of assets and liabilities over a given time period some basic relations among the main macroeconomic variables in an open economy; (ii) provide an introduction to the factors influencing international transactions and positions, and the extent to which such factors are sustainable; (iii) consider some of the implications of balance of payments financing and adjustments for economic policy.

It should be pointed out, however, that in this Section the linkages between an economy's domestic sectors and the rest of world are expressed as simple accounting identities from which no causal relation should be inferred. Though useful in describing those relationships, identities provide only a starting point for an analysis of the interaction among the main macroeconomic variables in an economic system. To draw casual relations, identities must be supplemented by specific hypotheses about the factors that determine the behavior of private agents and government sectors of the whole system. A deeper understanding of this approach will turn up as we discuss theoretical modelling in international finance in the next chapters.

5.2 Balance of Payments and National Accounts

In an open economy, the following two basic identities are commonly used to describes the balance between output, or aggregate supply, and aggregate expenditure, or use, of a country. The first defines gross domestic product (GDP) as

$$GDP \equiv C + I + G + X - \mathfrak{M}, \tag{5.1a}$$

where C =domestic consumption, I =domestic (private) investment, G =government expenditure, X =exports of goods and services, $\mathfrak{M}$ =imports of goods and services, and $\equiv$ a binary relation denoting equivalence, used to differentiate between identities, or accounting relations, and behavioral, or theoretical equations. Identity (5.1a) is the familiar expenditure approach to GDP.

The second defines gross national product (GNP) as

$$GNP \equiv GDP + NYB, \tag{5.1b}$$

where NYB =net income balance on primary and secondary income from abroad. The relation in (5.1b) is the income approach to GDP.

Substitution of (5.1a) in (5.1b) implies

$$Y \equiv C + I + G + X - \mathfrak{M} + NYB, \tag{5.2}$$

where Y is GNP.

If we now deduct taxes (T) from both sides of (5.2), we obtain

$$Y_d \equiv C + I + G - T + X - \mathfrak{M} + NYB, \tag{5.3}$$

where $Y_d \equiv Y - T$ denotes disposable income.

Balance-of-payments statistics shows that the current account balance (CA) is

$$CA \equiv X - \mathfrak{M} + NYB, \tag{5.4}$$

that is, the sum of balance on goods and services ($X - \mathfrak{M}$) and balance on income accounts (NYB). Thus, (5.3) can be rewritten as

$$Y_d \equiv C + I + G - T + CA. \tag{5.5}$$

As defined in the system of national accounts:

$$S^p \equiv Y_d - C \tag{5.6a}$$

$$S^g \equiv T - G \tag{5.6b}$$

$$S \equiv S^p + S^g, \tag{5.6c}$$

where S^p = private saving, S^g = public, or government saving, and S =national, or aggregate saving. Use of (5.6a)-(5.6c) in identity (5.5) then yields

$$CA \equiv S - I \equiv S^p + S^g - I \equiv (S^p - I) + (T - G)\,. \tag{5.7}$$

This identity points out that the current account reflects the gap between saving and investment of an economy, with a current account surplus indicating that national saving exceeds investment, or that private savings exceeds (private) investment and/or government budget is in surplus, and vice versa with a current account deficit. It is worthy to restate, however, that (5.7) is merely an identity that tells nothing about causation. Thus, it might be either that the current account deficit is the result of a shortage in private saving and/or a government budget deficits, or that the lack of national saving is due to the current account deficit. Put more simply, what identity (5.7) suggests is that any change in an economy's current account balance (e.g., a larger surplus or smaller deficit) must necessarily be matched by a change in saving relative to investment (e.g., an increase in domestic saving relative to investment). This highlights the relevance of understanding and assessing the effects on saving and investment of policy measures (e.g. changes in exchange rates, tariffs, etc.) designed to change the current account balance.

An alternative way of expressing the link between the external and internal sectors of an economy shown in identity (5.7) is as follows. Let $A \equiv C + I + G$ denote domestic absorption or expenditure and rewrite (5.2) as

$$Y \equiv A + CA.$$

It follows that

$$CA \equiv Y - A, \tag{5.8}$$

which states that the current account equals the difference between national income and absorption. The implication of this relationship is the same as that noted above and tells nothing about causality. It simply states that any changes in a country's current account (e.g., a larger surplus) require changes in domestic absorption relative to national income (i.e., a contraction in A, or an increase in Y). Hence, it would be inappropriate to use the identity (5.8) to analyze the impact of changes in Y or A on the current account balance without understanding the reaction of both private agents and the government to such changes.

As shown in the previous chapter and in Tab.1.3, the balance of payments statistics include not only the current account (i.e., the exchange of goods and services, and the receipt and payment of income and transfers) but also the capital and financial account (i.e., the flow of financial transactions involving changes in financial claims on, and liabilities to, the rest of the world). Also, the basic principle of double-entry bookkeeping implies that the sum of all international transactions—current, capital, and financial—is in principle equal to zero, though they may not balance in practice owing to errors or omissions. Assuming for simplicity that there are no recording errors or omissions, this balance between the financial account and the current and capital account can be expressed as

$$NLB \equiv CA + KA \equiv NFA, \tag{5.9}$$

or, alternatively, as

$$CA - (NKFA + \Delta FR) \equiv 0, \tag{5.10}$$

where NLB = net lending/net borrowing, KA = the capital account balance, NFA = net financial account, ΔFR = change in reserve assets, and $NKFA \equiv (NFA + KA) - \Delta FR$ =net capital and financial account (i.e., all capital and financial transactions excluding reserve assets).

The identity in (5.9) simply formalize the idea that in standard presentations of BP statistics net lending/net borrowing from the sum of the current account and capital account balances is conceptually equal to net lending/net borrowing from the financial account. The identity in (5.10) shows that the sum of all current, capital and financial account items, including reserve assets, is equal to zero. This means

$$CA \equiv NKFA + \Delta FR, \tag{5.11}$$

which states that that the net provision of resources to or from the rest of the world, as measured by the current account balance, must—by definition—necessarily be matched by a change in net capital and financial account balance plus foreign reserve assets. Thus, a surplus (deficit) in the current account is reflected in an increase (decrease) in net claims held by the private sector or government on nonresidents ($NKFA$) or an increase (decline) in official reserve assets (ΔFR), or both. From this perspective, the identity in (5.11) can be thought of as describing the budget constraint for the whole economy.

Because the above balance-of-payments accounting framework hold irrespective of the exchange rate regime adopted by a country, it is useful to consider the implications of the fundamental relationship described in (5.11) separately for fixed, flexible and managed exchange rates.

Fixed rates

Under a fixed or pegged exchange rate system, the country's monetary authorities intervene in the forex market to prevent exchange rate changes using its official reserves. Accordingly, transactions in reserve assets will be determined by the net demand or supply of foreign exchange at that exchange rate, namely

$$\Delta FR \equiv CA - NKFA.$$

This identity tells us that when exchange rate are fixed the change in official reserves equals the combined surplus/deficit difference in the current and capital account. Consequently, when $CA - NKFA > 0$ (a BP surplus) the net excess demand of the country's currency in the forex market will force the monetary authority to supply an amount of its currency equal to the excess demand, thereby purchasing foreign currency and creating $\Delta FR > 0$. Similarly, when $CA - NKFA < 0$ (a BP deficit) the monetary authority will buy whatever excess supply of its currency appear in the foreign market at the fixed rate, thereby selling foreign reserves and creating $\Delta FR < 0$.

In short, under a fixed-rate system the monetary authorities had to accommodate any change in the demand of their currencies if they were to prevent their exchange rates from changing. Therefore, ΔFR had to be equal to the combined balance on current and capital account $CA - NKFA$.

Flexible rates

Under a pure floating (truly flexible) exchange rate arrangement, there cannot be any change in official reserves because central banks do not intervene in the forex market to affect the valuation of the exchange rate. Accordingly, $\Delta FR = 0$ and

$$CA \equiv NKFA.$$

This identity shows that with flexible exchange rates, the (correctly measured) current account balance is exactly equal to the (correctly measured) capital and financial account balance, so that $BP \equiv CA - NKFA = 0$. What happens under a pure float regime is that any imbalance in a country's balance of payments ($BP \neq 0$) will automatically alter the exchange rate in the direction necessary to obtain $BP = 0$.

Therefore, a BP surplus ($CA - NKFA > 0$) should lead the exchange rate to appreciate, thereby making domestic goods and assets more expensive on international markets and applying downward pressures on the current account (and balance of payments) surplus. Similarly, a BP deficit should lead the exchange rate to depreciate, thereby causing improvements in CA and BP.

Managed rates

In the intermediate case of a managed float, purchases and sales of reserve assets by central banks are typically undertaken to achieve a desired exchange rate path for the domestic currency in terms of one or more foreign currencies. Accordingly, a BP imbalance should lead to changes in reserves assets, the exchange rate, or both so as to get $CA - (NKFA + \Delta FR) = 0$.

Balance of payments financing

The interrelationship between the current account and the capital and financial account can be seen more clearly by using the identities in (2.7) and (2.8), which state that the current account equals the difference between total domestic saving and investment or, alternatively, the gap between income and domestic expenditure. Hence, (2.11) we can be rewritten as

$$CA \equiv S - I \equiv Y - A \equiv NKFA + \Delta FR. \tag{5.12}$$

This identity describes the flows of resources and financial assets over time. It states that any gap between saving and investment or domestic income and expenditure, which is reflected in a current account imbalance, is matched by a change in private assets or official assets, or both. Thus, to the extent that domestic capital accumulation (I) or domestic absorption (A) is not matched by an increase in domestic saving (S) or domestic income (Y) there will be a current account deficit, which must be financed by a reduction in net foreign assets held by the private sector or government ($NKFA$) or a reduction in official reserve assets (ΔFR), or both. Obviously, the opposite comes about in the excess of saving case or current account surplus.

Nevertheless, if the gap between income and expenditure revealed not to be temporary or reversible, and hence persist over an extended period of time, then concerns about the country's external payments position are likely to materialize eventually, leading to abrupt and painful adjustments up to the possibility of a balance of payments crisis. This is because with $CA < 0$, for example, a country is financing the excess of consumption and investment over national income by using private and/or official assets or by borrowing abroad. This is not sustainable in the long run, because (*i*) the stock of reserve assets is limited and bound to be exhausted in a finite time, and (*ii*) the increase in liabilities (borrowing abroad) and/or the decline in foreign assets reduce the net flows of the income account. As a consequence, the current account deficits increases further and this can lead to a destabilizing situation in which the current account balance progressively worsens unless changes in economic policies (e.g., expenditure reducing policies) or adjustments in certain variables (e.g., exchange rates) are made to arrest the deterioration and avert a balance of payments crisis.

Problems may also arise when a country is facing a persistent current account surplus. In this case, the excess of aggregate saving over domestic investment or of income over expenditure is reflected in a rise in net foreign assets held by the private sector or a buildup of official reserve assets, or both. If expenditure expanding policies aimed at driving consumption toward foreign goods and away from domestic goods are not implemented, then important issue associated with financial stability and monetary policy management may arise for the domestic economy. In particular, when the surplus causes a buildup of reserve assets, the economy's monetary aggregates increase and a credit expansion will occur. If the credit expansion is too large and rapid so fuelling a credit boom, the economy may overheat, leading to inflationary pressures and/or vulnerabilities in the financial sector especially if weaknesses in financial sector supervision exist.

External vulnerability and IIP

As stressed in chapter 4, the international investment position account shows the value and composition of a country's international stock of assets and liabilities. It is closely connected to the flows of funds listed in the BP accounts, as the inflows and outflows of capital resulting from purchasing and selling of financial assets are added to and subtracted from the stocks of outstanding foreign assets and liabilities. Also, the IIP is a subset of the national balance sheet and equals the net worth of the economy, which is the balancing item of the national balance sheet. This is because in the aggregate the financial assets and liabilities of domestic sectors cancel each other out and a country's balance sheet consists of its stock of domestic nonfinancial assets plus its net IIP (the difference between external financial assets and liabilities).

The severe and recurrent financial crises experienced in many emerging and developed economies in the last 3 decades of increasing financial integration have brought the financial structure of economies - the composition and size of the assets and liabilities on the economy's financial balance sheet - to the fore. It as been observed that capital flows have been increasingly volatile with global market integration. Financial flows result to be high sensitive to market conditions, perceived policy weaknesses, and negative shocks. Capital inflows can suddenly revert following a decrease in confidence about the sustainability of a country's financial position, such as a high level of short-term debt, and this "sudden stop" in external financing often leads the economy to experience a severe financial crisis.

The modern literature on currency, banking and sovereign payments crises includes a set of models for understanding the role of the financial sector in the build-up to crisis vulnerability. These models are now collected under the heading of "balance sheet approach" to financial crises and provides a powerful analytical framework for exploring how balance sheet weaknesses in one sector can affect balance sheets in other sectors and give rise to a broader crisis. This approach focuses on some common types of balance sheet mismatches:

- Maturity mismatches, where a gap between short-term liabilities and long-term assets leaves an sector unable to honor its contractual commitments if creditors decline to roll over debt.

- Currency mismatches, where foreign-currency-denominated obligations and domestic-currency-denominated assets if unhedged leads heavy losses following a change in the exchange rate.

- Capital structure problems, where a heavy reliance debt financing rather than equity financing leaves a firm, a bank or a country less able to withstand revenue shocks.

- Solvency risk, where assets values are lower than liabilities value.

- Dependency problems, where a country is overexposed to another country and hence vulnerable to sudden changes about confidence in financial sustainability.

Therefore, because IIP data provide information on the composition of the financial structure of economies it also helps to signal the degree of vulnerability of the economy to changes in external market conditions. This is why it is a so critical indicator of financial (in)stability of a country.

Part III

The Theory of Exchange Rates

Chapter 6

Some Fundamental International Parity Conditions

In this chapter we look at two fundamental international financial relationships known as the *purchasing-power parity* (PPP) condition and the *interest parity* condition (IP).

6.1 The Purchasing Power Parity Principle

The purchasing-power parity (PPP) condition concerns the long-run relationship between the exchange rates and the price level and applies to goods and service markets. PPP is generally attributed to Gustav Casell's writings in the 1920s, although its intellectual origins date back to David Ricardo. The basic concept is that arbitrage forces will lead to equalization of goods prices internationally once the price of the goods are measured in the same currency. As such the theory represents an application of the *law of one price.*

This law simply states that in the presence of competitive international markets and in the absence of transport costs and other barrier to trade, identical products which are sold in different markets will have the same price when expressed in terms of a common currency.

Formally,

$$P_i = SP_i^*, \tag{6.1}$$

where P_i is the price of good i expressed in terms of home currency, S is the nominal exchange rate expressed as the price of foreign currency in terms of domestic currency, and P_i^* is the price of the same good expressed in foreign currency. Equation (6.1) is based upon the idea of perfect goods arbitrage. Arbitrage in the goods markets occurs where economic agents exploit price differences to gain a riskless profit. For example, if a TV set costs €1000 in Italy and the identical model costs $1200 in the US, then according to (6.1) the free-arbitrage exchange rate should be €1000/$1300=€0.8333/$1. Imagine the exchange rate is instead €0.850/$1, then it would pay for a US resident to purchase a TV set in Italy because with $1176.5 he would obtain €1000 which could be used to purchase a TV set in Italy, thus saving $23.5 compared to purchase in the US. Since agents will exploit this arbitrage possibility as long as the difference in price persists, an excess demand for euro and excess supply of dollars will start emerging in the forex market, leading the € to appreciate and the $ to depreciate. Such a process will continue until the euro-dollar rate reach the level of €0.8333/$1.

The proponents of PPP argue that the law of one price holds not only for individual goods, but also for identical bundles of goods. Hence, (6.1) can be extended to the n-goods case to obtain

$$P_t = S_t P_t^*, \tag{6.2}$$

where P is the aggregate price index of all goods and services sold in the domestic economy and expressed in the home currency (i.e., $P = \sum_{i=1}^{n} \alpha_i P_i$, with $\sum_{i=1}^{n} \alpha_i = 1$, and $\alpha_i =$ wheight of P_i in the aggregate index), and P^* is the aggregate price index of the same goods and services sold in the foreign country and expressed in terms of foreign currency (i.e., $P^* = \sum_{i=1}^{n} \alpha_i^* P_i^*$, with $\sum_{i=1}^{n} \alpha_i^* = 1$, and $\alpha_i^* =$ wheight of P_i^* in the aggregate index). Equation (6.2) is known as the *absolute version* of PPP. It states that a rise in the domestic price level relative to the foreign price level will lead to a depreciation of the domestic currency against the foreign currency and vice versa, and can be easily understood by rewriting (6.2) as

$$S_t = \frac{P_t}{P_t^*}. \tag{6.2a}$$

Equation (6.2) is unlikely to hold exactly in the real world where transport costs, imperfect information and distorsing effects of tariffs and other form of protectionism may introduce marked deviations from PPP, especially in the short-run. Nevertheless, proponents of PPP argue that a weaker form known as *relative purchasing-power parity*

(RPPP) can be expected to hold even in the presence of such distortions. This version can be obtained from (6.2) by taking logs and differentiating with respect to time. Hence,

$$\dot{p}_t = \dot{s}_t + \dot{p}_t^*,$$

whence

$$\dot{s}_t = \dot{p}_t - \dot{p}_t^*, \tag{6.3}$$

where $\dot{s}_t = (d \ln S_t/dt) = (1/S_t)(dS_t/dt)$ is the rate of change in the exchange rate, $\dot{p}_t = (d \ln P_t/dt) = (1/P_t)(dP_t/dt)$ is the domestic inflation rate, and $\dot{p}_t^* = (d \ln P_t^*/dt) = (1/P_t^*)(dP_t^*/dt)$ is the foreign inflation rate. This equation tells that the rate of variation in the exchange rate reflects the differential between the domestic inflation rate $\dot{p}_t$ and the foreign inflation rate $\dot{p}_t^*$. If, for example, the inflation rate in EU is 4% whilst that in the US is 2%, the euro-dollar exchange rate should be expected to depreciate by 2%. Thus, according to (6.3) the countries with higher inflation rate will tend to experience a continuous depreciation of their currencies in the forex markets.

Notice that (6.2) can be used to obtain a measure of the real exchange rate (Q) defined in terms of the nominal exchange rate adjusted for the relative price levels in the home and foreign country (see, Chap.2, Sect. 2.4, eq. 1.6). Hence,

$$Q_t = \frac{S_t P_t^*}{P_t},$$

whence, applying logs and differentianting with repect to time, we get

$$\dot{q}_t = \dot{s}_t + \dot{p}_t^* - \dot{p}_t, \tag{6.4}$$

which expresses the movements in the real exchange rate in terms of changes in $\dot{s}_t$, $\dot{p}_t^*$, and $\dot{p}_t$. Therefore, if the movements in the nominal exchange rate equals the differential between the home and foreign inflation rate - that is if (6.3) holds -, then the real exchange rate does not change and $\dot{q}_t = 0$; otherwise, $\dot{q}_t < 0$ $\left(\dot{q}_t > 0\right)$ if $\dot{p}_t > \dot{p}_t^*$ $\left(\dot{p}_t < \dot{p}_t^*\right)$ and the real exchange rate appreciates (depreciates).

6.2 The Interest Parity

The purchasing-power parity condition is concerned with arbitrage in goods and services markets and has noting to say about arbitrage in international capital markets. For these markets, an important parallel condition exists that emphasize the role of interest rates. It is known as the *interest parity* conditions and applies to arbitrage in international financial markets. This condition simply states that, when measured in a common currency, interest yields on identical financial assets should be the same everywhere. If this were not true, then funds will tend to flow out or in from one country to other countries until indifference in investing at home or abroad prevails.

We explored a version of this condition, known as *covered interest parity* (CIP) in chapter 2, section 2.3, when we addressed the issue of how the forward market can be used to eliminate exchange-rate risk and exposure in interest arbitrage operations. For convenience, the CIP condition (eq.1.5) is reported in the following equation:

$$\frac{\mathcal{F}_{t,t+1}}{S_t} = 1 + f_p = \frac{1+i_t}{1+i_t^*}$$

or

$$i_t = i_t^* + f_p, \tag{6.5}$$

where $\mathcal{F}_{t,t+1}$ is the one-period forward exchange rate, S_t is the spot exchange rate, and $f_p = (\mathcal{F}_{t,t+1} - S_t)/S_t$ the forward margin. Equation (6.5) states that international investors should be indifferent between domestic- and foreign-currency denominated securities when the domestic-currency interest rate equals the foreign-currency rate plus the forward margin.

Equation (6.5) is the condition for hedged or covered activity in the money markets because it involves the forward market. It can be argued that a similar *unhedged* parity condition should also hold. This condition involves expectations about the future value of the exchange rate. To see, let us assume there is perfect capital mobility and financial assets are perfect substitutes. Assume also that agents hold perfect (or deterministic) exchange rate expectations, that is, are able to predict exactly the future value of the spot exchange rate.[1] Alternatively, we can assume that agents are risk-neutral, namely they care only about the yield of his funds and not about risk. Imagine now they face the following two alternatives:

[1] This is identical to the rational expectations hypothesis in a deterministic world. Rational expectations is a powerful way of modelling expectations under uncertainty. It means that agents use economic theory (i.e., the model) to obtain forecasts of the future value of the relevant economic variables that are optimal given the available information set and model structure. This assumption does not imply that agent's forecasts may not be wrong, rather that they are on average correct and do not reveal persistent errors over time. The hypothesis of rational expectations was originally advanced by John Muth and became influential when Robert Lucas and other proponents of the New Classical theory applied it to macroeconomic analysis and policy strategies evaluation. See, e.g., Muth (1961), Shiller (1978), Sargent (1987).

- investing their funds at home, earning an interest rate equal to i_t, or
- converting them into foreign currency at the spot exchange rate S_t, investing them abroad and earning an interest rate equal to i_t^*, and converting them back into domestic currency at the expected spot exchange rate $S_{t,t+1}^e$.

It follows that agents will be indifferent between the two alternative when

$$S_t(1+i_t) = S_{t,t+1}^e(1+i_t^*),$$

or

$$\frac{S_{t,t+1}^e}{S_t} = \frac{1+i_t}{1+i_t^*} \tag{6.6}$$

where $S_{t,t+1}^e$ = expected spot exchange rate held at time t for time $t+1$.

Letting $\left(S_{t,t+1}^e/S_t\right) = 1+\Delta s_t^e$, where $\Delta s_t^e = (S_{t,t+1}^e - S_t)/S_t$ denote the expected rate of change in S_t, we can rewrite (6.6) as

$$1+\Delta s_t^e = \frac{1+i_t}{1+i_t^*},$$

whence, taking logs of both sides, we get

$$\Delta s_t^e = i_t - i_t^*$$

or, alternatively

$$i_t = i_t^* + \Delta s_t^e. \tag{6.7}$$

This equation is known as the *uncovered interest parity* (UIP) condition. It states that international investors should be indifferent between domestic- and foreign-currency denominated securities when the domestic-currency interest rate equals the foreign-currency rate plus the expected rate of change of the spot exchange rate.

By comparing (6.5) and (6.7) we observe that covered and uncovered interest parity are equivalent if $\Delta s_t^e = (S_{t,t+1}^e - S_t)/S_t$ equals $f_p = (\mathcal{F}_{t,t+1} - S_t)/S_t$ or $\mathcal{F}_{t,t+1} = S_{t,t+1}^e$, namely if the forward exchange rate is equal to the expected future spot exchange rate. This is what is expected to occur if speculators are risk-neutral and transaction costs are absent, i.e., capital mobility is perfect. To see, imagine, for example, that speculators expect the euro to be trading at €0.850/$1 in 1-year's time and the forward rate for 1 year is €0.820/$1. Speculators then would buy the dollar forward for €0.820 and expect to get €0.03=€0.850-€0.820 on each dollar when the dollars are sold at €0.850 each. The forward buing of the dollar would drive up the dollar forward rate, and this process would continue until the forward price is no longer below the expected spot price, that is, up to

$$\mathcal{F}_{t,t+1}\,(€/\$) = S^e_{t,t+1}\,(€/\$)\,.$$

Hence, speculation activity in foreign exchange market will tend to make the forward exchange rate to be approximately equal to the expected exchange rate. It is worthy to restate, however, that the above equality requires perfect capital mobilty and risk neutrality. Thus, in general, we cannot expect UIP to hold.

The interest parity conditions discussed so far involved only the nominal interest rates. Nevertheless, using equations (6.3) and (6.7), it is possible to get an analogous parity condition in terms of real interest rates. To see, observe that under perfect foresight, agents are able to predict the exact future value of the spot exchange rate. Thus, $S^e_{t+1} = S_{t+1}$, so that

$$\Delta s_t = \left[(S_{t+1} - S_t)/S_t\right] = \Delta s^e_t = \left[(S^e_{t,t+1} - S_t)/S_t\right]. \tag{6.8}$$

Given (6.8), rewrite the RPPP condition in a discrete time environment as

$$\Delta s^e_t = \Delta s_t = \Delta p_t - \Delta p^*_t \tag{6.9}$$

where $\Delta p_t = \left[(P_{t+1} - P_t)/P_t\right]$ and $\Delta p^*_t = \left[\left(P^*_{t+1} - P^*_t\right)/P^*_t\right]$ are the domestic and foreign inflation rate, respectively.[2] Substituting (6.9) into (6.7), leads to

$$i_t = i^*_t + \Delta p_t - \Delta p^*_t,$$

which can be rearranged to yield

$$i_t - \Delta p_t = i^*_t - \Delta p^*_t. \tag{6.10}$$

[2] To derive (6.9) notice that leading (6.2) one period

$$P_{t+1} = S_{t+1} P^*_{t+1}.$$

If we now divide through by $P_t = S_t P^*_t$, we get

$$\frac{P_{t+1}}{P_t} = \left(\frac{S_{t+1}}{S_t}\right)\left(\frac{P^*_{t+1}}{P^*_t}\right),$$

or

$$1 + \Delta p_t = (1 + \Delta s_t)\,(1 + \Delta p^*_t)\,.$$

Next, taking logs of both sides, we have

$$\Delta p_t = \Delta s_t + \Delta p^*_t,$$

whence

$$\Delta s_t = \Delta p_t - \Delta p^*_t.$$

This equation equals the (nominal) interest rate less the expected inflation rate in the domestic and foreign countries. According to the well-known Fisher definition, the interest rate minus the expected inflation rate is the real interest rate. As a result, (6.10) is called the Fisher-open condition or the *real interest parity* (RP) condition. It states that real interest rate in different countries tend to be equal under PPP and UIP condition.

Chapter 7

Exchange Rate and Balance of Payments

7.1 Introduction

In this chapter we look at the relationships between the exchange rate and the balance of payments. We first discuss the relationship between the exchange rates and the current account, thus focusing on variables describing the real side of an economy and of the balance of payments. We then extend the focus to the monetary side and the capital account of the balance of payments to include the relationships between the exchange rates and the interest rates.

In analyzing the relationship between the exchange rate and the current account we can distinguish three main approaches: the elasticity approach, the absorption approach, and the multiplier approach. Let us start with the first approach.

7.2 The Elasticity Approach

This approach provides a simple analytical framework to examine the effects on the current account of a variation in the exchange rate. The approach builds on a simple model that takes the domestic and foreign price as well as all other variables that might influence the balance of payments to be fixed so that changes in relative price are caused by changes in the nominal exchange rate. Thus, it performs a partial equilibrium analysis where the *ceteris paribus* clause is imposed when the exchange rate varies. To see, assume for simplicity, that income flows and unilateral transfers are equal to zero, so that the current account is equal to the trade account. Accordingly, when expressed in terms of domestic currency, the current account balance is given by:

$$CA = P_x X - S P_{\mathfrak{M}} \mathfrak{M}, \tag{7.1}$$

where P_x is exports price (in terms of domestic currency), X is the volume of exports, S is the nominal exchange rate (domestic currency units per unit of foreign currency), $P_{\mathfrak{M}}$ is imports price (in terms of foreign currency), and $\mathfrak{M}$ the volume of imports. Assume also that X changes in the same direction of S, i.e., increase when S increase (depreciate) and decrease when S (decrease), whereas $\mathfrak{M}$ changes in the opposite direction of S, i.e., decrease when S increase and vice versa.[1] Setting, for simplicity, $P_x = P_{\mathfrak{M}} = 1$, (7.1) becomes:

$$CA = X - S\mathfrak{M}. \tag{7.2}$$

Compute next the total differential of (7.2) to attain

$$dCA = dX - \mathfrak{M} dS - S d\mathfrak{M}, \tag{7.3}$$

and divide through by dS to yield

$$\frac{dCA}{dS} = \frac{dX}{dS} - \mathfrak{M} - \frac{S d\mathfrak{M}}{dS}. \tag{7.4}$$

Let

$$\eta_x = \frac{dX}{dS}\frac{S}{X} \text{ and } \eta_m = -\frac{d\mathfrak{M}}{dS}\frac{S}{\mathfrak{M}} \tag{7.5}$$

be the elasticity of export and of imports over the exchange rate, respectively. It follows that

$$dX = \eta_x \frac{dS}{S} X \text{ and } d\mathfrak{M} = -\eta_m \frac{dS}{S}\mathfrak{M}. \tag{7.6}$$

Substitution of (7.5) and (7.6) into (7.4) yields

$$\frac{dCA}{dS} = \eta_x \frac{X}{S} - \mathfrak{M} + \eta_m \mathfrak{M},$$

[1] This is true under the ceteris paribus clause, which takes all the other relevant variables (price, incomes, etc.) fixed, and the implicit assumption that the domestic and foreign supplies of goods and services are perfectly elastic. See, e.g., Gandolfo (2002), pp. 82-83.

and dividing through by $\mathfrak{M}$

$$\frac{dCA}{dS}\frac{1}{\mathfrak{M}} = \eta_x \frac{X}{S\mathfrak{M}} - 1 + \eta_m. \tag{7.7}$$

Assuming finally that the current account is initially in equilibrium, $(X/S\mathfrak{M}) = 1$ and (7.7) can be rearranged to yield

$$\frac{dCA}{dS} = \mathfrak{M}\left(\eta_x + \eta_m - 1\right). \tag{7.8}$$

Equation (7.8) implies that

$$\frac{dCA}{dS} > 0 \text{ iff } \quad (\eta_x + \eta_m - 1) > 0 \tag{7.9}$$

and is known in the literature as the Marshall-Lerner condition, from the seminal contribution of Marshall (1923) and Lerner (1944) on international trade.[2] Equation (7.9) simply states that starting from an initial equilibrium in the current account, a devaluation of the exchange rate will improve the balance (i.e., $dCA/dS > 0$) only if the sum of exchange-rate elasticities of exports and imports is greater than one, that is, if $\eta_x + \eta_m - 1 > 0$. If this were not the case, then a devaluation would lead to a deterioration and not to an improvement in CA.

The economic intuition behind (7.9) is straightforward. Following a devaluation of a currency, there are two effect in play: a price effect and a volume effect. The *price effect* comes from the fact that a rise in S (a devaluation) makes imports more expensive in terms of domestic currency and exports cheaper in terms of foreign currency. Therefore, the price effect contributes to a worsening in the current account. The *volume effect* comes from the fact that the volume of exports should increases and the volume of imports decreases following the rise in S. The volume effects clearly contributes to improving the current account.

As consequence, the net effect depends on whether the price or volume effects dominates. If exports and imports elasticities are high enough to satisfy the Marshall-Lerner condition (7.9), the volume effects dominates and the current account improve following a nominal depreciation of the exchange rate. Conversely, if imports and exports demand are inelastic or do not satisfy condition (7.9), the price effects dominates and the current account worsens.

[2] As noted in Gandolfo (2002), however, condition (7.9) is to be found in earlier contributions by Robinson (1937) and Bickerdicke (1920), and should also be named as the Bickerdicke-Robinson condition.

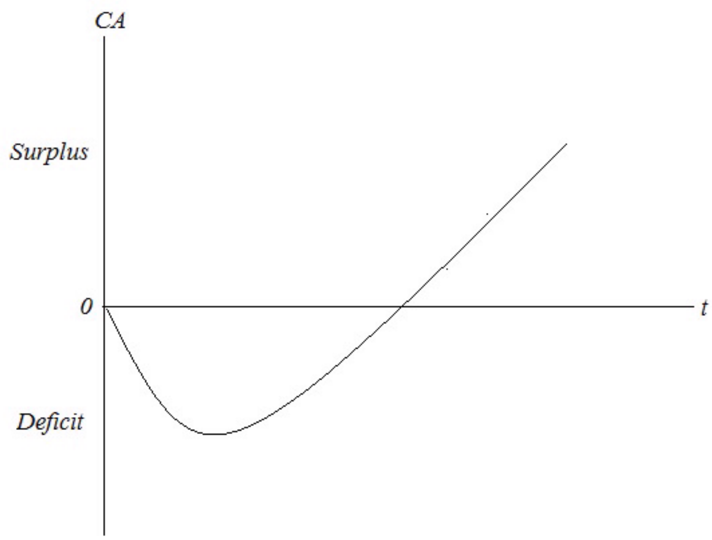

Figure 7.1 The J-curve effect.

The available empirical evidence points to show that conditions (7.9) tends to be satisfied in many developed countries and a number of developing or emerging countries.[3] It also shows that the Marshall-Lerner condition is met in the long run but much less in the short run, so giving rise to the possibility that a nominal depreciation of the exchange rate may lead to a (temporary) worsening rather than improvement in the current account balance. Indeed, if imports and exports demand are more inelastic in the short run than the long run, then following a depreciation we may find that a country experiences a worsening in the current account balance in the short run but an improvement afterwards. This phenomenon is known as the *J-curve* effect. The J-curve effect is illustrated in figure 7.1. The figure assumes that a devaluation occurs at time 0, and that because of short-run rigidities people tend to spend temporarily more on imports than the increase in exports. As a consequence, the current account balance worsens immediately following the nominal depreciation to improve only later when imports and exports elasticities start increasing.

[3] This is not a surprise since emerging and developing economies are more dependent on imports than developed countries and hence tend to show more inelastic imports demand.

The main reasons behind the J-curve phenomenon are as follows:

- *Time lag in consumers' preference adjustment.* It takes time for consumers to adjust their preferences toward substitutes, that is, switching away from foreign imported goods to domestically produced goods.

- *Time lag in producers response.* It takes time for producers to expand production of exportables (inelastic domestic supply curve).

- *Imperfect competition.* To preserve a share in foreign markets, foreign exporters may respond to a depreciation by reducing their exports price. Conversely, if imports are used as inputs for exporting industries, domestic exporters may be forced to rise the price of their goods, thus reducing the competitive advantage of the nominal depreciation.

7.3 The Absorption Approach

The elasticity approach considers the effects of a change in S on prices and volumes of export and import under the assumption that all other things are equal (the ceteris paribus clause). However, the changes in exports and imports brought about by a nominal depreciation will affect the national income, which in turn will necessarily impact on aggregate demand and the current account. Thus, ignoring these effects when investigating the impact of nominal depreciations might reveal a serious shortcoming. A more comprehensive analysis including those effects is therefore needed and this may be found in the so called *absorption approach* to the current account.

To see, let us turn on equation (5.8) that we rewrite below for convenience

$$CA = Y - A, \tag{7.10}$$

which states that the current account (CA) reflects the gap between domestic output (Y) and domestic absorption (A). Computing the effect on CA of a change in S leads to

$$\frac{dCA}{dS} = \frac{dY}{dS} - \frac{dA}{dS}. \tag{7.11}$$

Equation (7.11) shows that $(dCA/dS) \gtreqless 0 \Longrightarrow (dY/dS) \gtreqless (dA/dS)$, which means that the current account balance will improve, deteriorate, or stay unchanged, following a devaluation, according to whether income will rises more, less or the same as absorption. Hence, understanding how a rise in S (a nominal depreciation) will affect both Y and A is central to the absorption approach.

For this purpose notice that $A \equiv C + I + G$ and that C and I are functions of income, so that when Y changes A will also changes following the changes in both C and I. This means that we can write the effect on absorption of a change in S as

$$\frac{dA}{dS} = \alpha \frac{dY}{dS} + \frac{dA^D}{dS}, \tag{7.12}$$

where $\left(dA^D/dS\right)$ is the direct effect, $(\alpha dY/dS)$ is the indirect effect through the change in income, and α is the marginal propensity to absorb equals to the sum of the marginal propensity to consume and the marginal propensity to invest. Substituting (7.12) into (7.11) yields

$$\frac{dCA}{dS} = (1-\alpha)\frac{dY}{dS} - \frac{dA^D}{dS}, \tag{7.13}$$

which partitions the total effect of a devaluation into three parts:

- (dY/dS), which measures the direct impact on income;
- $\left(dA^D/dS\right)$, which measures the direct impact on absorption; and
- $(\alpha dY/dS)$, which measures the indirect effect on A driven by the change in income.

Equation (7.13) reveals that the condition for a nominal depreciation to improve the current account balance is

$$(1-\alpha)\frac{dY}{dS} > \frac{dA^D}{dS}, \tag{7.14}$$

which says that $(dCA/dS) > 0$ if and only if the total change in income exceeds the direct impact on absorption. Thus, understanding the factors that drive the changes in Y and A and their size after a devaluation is a key issue within the framework of the absorption approach. A synthetic overview of the main factors at play is as follows.

Effects on income

These include two main effects: an idle-resouce effect, a terms of trade effect.

Idle-resouce effect. If there are idle resources, the effect of a rise in S (devaluation) will be to increase Y as demand is switched to home-produced goods. How much Y will increase depends on the degree of import substitution, the propensity of other countries to import, and the size of foreign multiplier (see, Sect. 7.3 below). From (7.14) we see, however, that the idle resource effect will improve CA, only if $\alpha < 1$. If $\alpha > 1$, the current account balance will worsen.

Terms of trade effect. This effect flows from the change in terms of trade brought about by a rise in S. Terms of trade are expressed as the price of exports divided by the price of imports, namely

$$\mathcal{T} = \frac{P^X}{SP^{\mathfrak{M}}},$$

where $\mathcal{T}$ denotes terms of trade, and P^X and $P^{\mathfrak{M}}$ are the price of exports and imports, respectively. Hence, when S rise, $SP^{\mathfrak{M}}$ will also rise. If this is not matched by a corresponding rise in $P^{\mathfrak{M}}$, then the terms of trade deteriorate. The fall in $\mathcal{T}$ represents a loss of real income as more exports are required to exchange for one unit of imports, and this reduce income.

As a result, the effect of a rise in S on the income of the devaluing country is ambiguous. Even if net exports earnings increase after a devaluation assuming the Marshall-Lerner condition (7.9) is fulfilled, the negative terms of trade effects works to reduce real income, so that the net effects is a priory uncertain.

Effects on absorption

These include at least three relevant effects: a real balance effect, an income redistribution effect, a money illusion effect.

Real-balance effect. This effect comes about through the desire of agents to hold a constant proportion of their real income in the form of real money balances, namely

$$\frac{M}{P} = \kappa Y,$$

where (M/P) denotes real money balances, M is the nominal stock of money, P is an aggregate price index, and κ a positve constant. If the value of real money balances is eroded by a rise in price after a devaluation (recall that $P = SP^*$ under PPP), agents will attempt to restore the real value of money balances by rising M, which can be done by reducing their expenditure out of real income, and hence by reducing absorption.

Income redistribution effect. The increase in price will also lead to a redistribution of income (e.g., from those with fixed income, who generally have a low marginal propensity to save, to those with variable income, who have a higher marginal propensity to save), and this will affect absorption.

Money illusion effect. This effect, which may occur in the short run, arises when agents fail to adjust their money expenditure in the same proportion to the rise in prices, even though the real income has not changed, that is, even though price and nominal income increased in the same proportion.

The other minor forces affecting absorption concern the *expectations effect*, which may prompt agents to increase their expenditures in advance to avoid paying higher prices in the future; the *import-price effect*, which comes about through the effects on imported investment goods and foreign goods in general caused by the change in their price.

Overall, the absorption approach points out that a nominal depreciation will have many different and often conflicting effects on the current account, so it is extremely difficult to predict whether it will improve or worsen the account.

7.4 The Multiplier Approach

The multiplier approach can be seen as an alternative analytical framework to analyze balance-of-payments adjustment when the exchange rate and prices are rigid or fixed. It involves automatic adjustment that operates on the current account via changes in national income. The mechanism was introduced by Harrod (1933) before the Keynesian theory of multiplier, and popularized by other Keynesian followers, notably by Metzler (1942), Machlup (1943), and Johnson (1956), who applied it to an analysis of open economies. The most straightforward way of describing the income adjustment mechanism is to use a standard Keynesian income-expenditure model extended to include the foreign sector. The model consists of the following equations:

$$E \equiv C + I + G + X - \mathfrak{M} \tag{7.15}$$

$$C = C_0 + cY_d \tag{7.16}$$

$$Y_d \equiv Y - T \tag{7.17}$$

$$T = T_0 + \tau Y \tag{7.18}$$

$$I = I_0 + hY \tag{7.19}$$

$$X = X_0 \tag{7.20}$$

$$\mathfrak{M} = \mathfrak{M}_0 + \mu Y \tag{7.21}$$

$$Y = E, \tag{7.22}$$

where E = aggregate spending on domestically produced goods and services, G = government spending on domestic goods and services, and the other variables have the same meaning as before.

Equation (7.15) defines aggregate spending. Equation (7.16) describes the consumption function, where C_0 represents the autonomous component (i.e., the part of C that does not depend on income) and c $(0 < c < 1)$ is the marginal propensity to consume. Equation (7.17) defines disposable income. Equation (7.18) is the tax function, where T_0 is the autonomous component and τ $(0 < \tau < 1)$ is the marginal propensity to tax. Equation (7.19) is the investment function, where I_0 is the autonomous component and h $(0 < h < 1)$ is the marginal propensity to invest. Equation (7.20) states that exports are exogenously fixed at X_0. Equation (7.21) is the import function, where $\mathfrak{M}_0$ is the autonomous component and μ $(0 < \mu < 1)$ is the marginal propensity to import. Equation (7.22) is the equilibrium condition on the goods market.

In order to discuss how automatic adjustment via income changes works, let us first solve the model for the equilibrium value of national income. This requires substitution of equations (7.15)-(7.21) into (7.22) to yield

$$Y = \frac{1}{1 - c(1-t) - h + \mu}(C_0 + I_0 + X_0 + G - T_0 - \mathfrak{M}_0), \qquad (7.23)$$

where $[1 - c(1-t) - h + \mu] > 0$ for the solution to be economically meaningful. Equation (7.23) expresses the equilibrium value of national income, Y, as a function of all exogenous and autonomous variables affecting aggregate spending E. The factor $1/\left[1 - c(1-t) - h + \mu\right]$ is known as the open-economy multiplier and marks out the effects on Y of a change in the autonomous expenditure on domestic and foreign goods. Notice that the open-economy multiplier is greater than 1 but lesser than that for the closed economy. This is because: (i) under $[1 - c(1-t) - h + \mu] > 0$, $[c(1-t) + h] < 1 + \mu$ or $[c(1-t) + h - \mu] < 1$, so that $1/\left[1 - c(1-t) - h + \mu\right] > 1$; and (ii) the additional leakage due to imports (μ) shortens its size below $1/\left[1 - c(1-t) - h\right]$, which marks out the closed-economy multiplier.

To see now how balance-of-payments adjustment problems can be analyzed under the multiplier approach, let us assume that the current account is initially in equilibrium and that exports increase exogenously of ΔX_0.[4] Adding to the above model the equation

$$CA = X_0 - \mathfrak{M}, \qquad (7.24)$$

which defines the current account balance under the assumption that the prices and the exchange rate are fixed and normalized to one, we can

[4] It is worth recalling that the model described in Eqs. (7.15)-(7.22) and the assumption that the income account is always in balance, implies $BP = CA$.

see that the increase in exports shifts initially CA from a 0equilibrium to a surplus position of

$$\Delta CA = \Delta X_0.$$

However, as shown by equation (7,23), the increase in exports brings about an increase in income of

$$\Delta Y = \frac{1}{1 - c(1-t) - h + \mu}\Delta X_0,$$

which in turn determine an increase in imports of

$$\Delta \mathfrak{M} = \mu \Delta Y = \frac{\mu}{1 - c(1-t) - h + \mu}\Delta X_0.$$

This increase in imports tends to offset the initial positive effects of exports on the current account balance, so that, using (7.24), the net effect on CA can be computed as

$$\begin{aligned} \Delta CA &= \Delta X_0 - \Delta \mathfrak{M} = \Delta X_0 - \frac{\mu}{1 - c(1-t) - h + \mu}\Delta X_0 \implies \\ \Delta CA &= \frac{1 - c(1-t) - h}{1 - c(1-t) - h + \mu}\Delta X_0. \end{aligned} \tag{7.25}$$

Equation (7.25) tells us that the automatic adjustment in the current account works via the changes in income and in imports brought about by the initial change in exports. If $[1 - c(1-t) - h] > 0$ and $[1 - c(1-t) - h + \mu] > 0$, $[1 - c(1-t) - h + \mu] > [1 - c(1-t) - h]$, and $0 < \Delta CA < \Delta X_0$. As a result, the adjustment process through the income multiplier will not be complete, and the effect on CA of the initial change in exports will persists.

Notice that using Eqs. (7.21) and (7.23) we can rewrite (7.24) in difference form as

$$\begin{aligned} \Delta CA &= \Delta X_0 - \Delta \mathfrak{M} = \Delta X_0 - \Delta \mathfrak{M}_0 - \mu \Delta Y \implies \\ \Delta CA &= \Delta X_0 - \Delta \mathfrak{M}_0 - \frac{\mu(\Delta C_0 + \Delta I_0 + \Delta X_0 + \Delta G - \Delta T_0 - \Delta \mathfrak{M}_0)}{1 - c(1-t) - h + \mu} \end{aligned} \tag{7.26}$$

and use it to compute the effects on the current account of a change in any autonomous expenditure. In particular, we can compute the effects of an increase in government spending, which is given by

$$\Delta CA = -\frac{\mu}{1 - c(1-t) - h + \mu}\Delta G. \tag{7.27}$$

This equation reveals that an increase in government spending leads to a deterioration of the current account balance, and describes a phenomenon known as *twin deficits* by which a rise in G leads to a deterioration in both the government budget balance and the current account balance[5].

[5] See, e.g., Obstefeld (1989), Turnovsky-Sen (1991), Kaway-Maccini (1995), Piersanti (2000, 2002), Bussière et al. (2005).

7.5 The Interest Rates and the Capital Account

Sections (7.2)-(7.4) focused on adjustments in the current account component of the balance of payments. However, the BP account also includes a section on capital and financial transactions which has become increasingly relevant with global financial markets integration. As seen in chapters (4)-(6), the amount of capital flowing into or out of a country depends on the rates of return in the country relative to rates of return elsewhere, as well as on relative risks. Hence, we should expect that, *ceteris paribus*, a rise in a country's interest rate or expected profits tends to increase the inflows of capital and similarly a decline in expected returns tends to reduce the inflows and/or increase the outflows of capital. These effects follow from the interest parity conditions we have discussed in chapter 6 and reveal an automatic adjustment mechanism that works though changes in the interest rates and the money supply brought about by balance of payments disequilibria. To see observe first that combining identities (5.9), (5.10) and (5.11) we can write the equation

$$BP \equiv CA + KA = \Delta FR, \tag{7.28}$$

which states that BP imbalances will be matched by changes foreign reserves assets (ΔFR) held at the central bank. Hence, under fixed or managed exchange rate systems, a surplus in the balance of payments will lead to an increase in official foreign reserves and a deficit to a decline in ΔFR.[6]

From the balance sheet of the central bank, ignoring net worth, we can write the money supply or money stock in the economy (M^s) as

$$M^s \equiv M^H + FR. \tag{7.29}$$

This identity says that a country's money stock is made up of two component: the domestic component or domestic credit $\left(M^H\right)$, that is lending to the private sector (by commercial banks) and to the government (by the central bank), plus the country's official foreign reserves (FR) valued in the domestic currency. Transforming (7.9) into difference form yields

$$\Delta M^s \equiv \Delta M^H + \Delta FR, \tag{7.30}$$

which shows that any change in the money supply can come about either through a change in M^H, or a change in FR. The identity (7.30)

[6] Conversely, under pure floating, $\Delta FR = 0$ and BP disequilibria will be cleared by variations in the exchange rate. See Chap. 5, Sect. 5.2

means that the domestic money supply cannot be fixed exogenously by the central bank, as it is endogenously determined by movements in official foreign reserves coming from BP imbalances. Thus, even assuming that the central bank has got full control over domestic credit movements and sets, say, $\Delta M^H = 0$, the money stock in the economy can equally change if $\Delta FR \neq 0$. In other words, changes in official foreign reserves will induce changes in the money supply even when the domestic credit component is kept unchanged, so that under $\Delta M^H = 0$, $\Delta M^s > 0$ if $BP > 0$ and hence $\Delta FR > 0$, and similarly $\Delta M^s < 0$ when $BP < 0$ and $\Delta FR < 0$.

The induced changes in the money supply will affect the interest rates, which in turn affect national income, expenditure and the current account. In addition, interest rate movements will also affect the capital flows and the capital account. There is, therefore, an additional adjustment mechanism that operates on the balance of payments via in the money supply and the interest rates. This mechanism works as follows. If a surplus in the balance of payments occurs, because of a combined surplus in the current an capital account as shown in Eq. (7.28), then $\Delta FR > 0$. If this effect is not sterilized (see, Chap.), the domestic money supply will increase. The money-supply increase lowers the domestic interest rates, stimulating investment, which in turn raise national income, consumption and imports, thereby helping reduce the current account surplus. In addition, the interest-rate decline will make investment in domestic tradable financial assets (securities and so on) appear relatively less attractive compared to those in the other countries. As a result, funds will start flowing out of the home country, thereby affecting the capital account and correcting the original surplus. Clearly, a reverse adjustment process operates if a deficit in the balance of payments occurs.

It is worth repeating, however, that the above automatic adjustment mechanism operates under fixed or managed exchange-rate regimes. If the country operates under a pure flexible-rate regimes, then BP adjustments will work via changes in the exchange rates.

Chapter 8

Models of Exchange Rate Determination

8.1 Introduction

In Chapters (2) and (6) we set out a very simple model of exchange rate determination and discussed some parity conditions that are widely used in the literature. Nevertheless, they are far from a satisfactory explanation of observed exchange-rate behavior. The reason for this disappointment is simple. The model discussed in chapter 1 is too churlish for a full understanding of the forces that determine the exchange-rate and, in particular, its equilibrium value. The arbitrage conditions examined in chapter (6) describes only equilibrium conditions, and do not imply any causal relationship between the variables involved. To go beyond this shortcomings, in this chapter we discuss some more sophisticated exchange-rate models that have been advanced in the literature to analyze exchange-rate behavior more cogently.

8.2 The Monetary Model

The first model we explore is known as the *flexible-price monetary model*. The model is based on the following key assumptions:

- No barriers to trade, perfect capital mobility and perfect substitutability between domestic and foreign bonds, so that purchasing power parity (PPP) and uncovered interest parity (UIP) hold at all times.

- Full price and wage flexibility, so that domestic output and labour market tend always to gravitate towards the full employment or flexible price equilibrium level of output.

- Perfect foresight, which is the equivalent of rational expectations in a deterministic world. This hypothesis means that agents are able to predict exactly the future value of the relevant economic variables.

The monetary approach to exchange rate determination has a long history and can be dated back to Cassel's writings in the period 1919-30. Promoted in the 1960s by Robert Mundell (1968) and Harry Johnson (1972), emerged as the dominant exchange rate model in the 1970s (see Frenkel and Johnson, 1978). In the mid-1970s and early 1980s it was extended to include rational expectations and short-term overshooting (Dornbusch, 1976) and to allow for multiple traded goods and real shocks across countries (Stockman, 1980; Lucas, 1982).

Most of the main features of the monetary approach to exchange-rate determination can be illustrated by the following set of equations:

$$\frac{M_t}{P_t} = Y_t^{\alpha_1} e^{-\alpha_2 i_t} \tag{8.1}$$

$$P_t = S_t P_t^* \tag{8.2}$$

$$e^{i_t} = e^{(i_t^* + \Delta s_{t+1}^e)} \tag{8.3}$$

$$\frac{M_t^*}{P_t^*} = Y_t^{*\alpha_1} e^{-\alpha_2 i_t^*}, \tag{8.4}$$

where M_t = date t domestic nominal money stock, M_t^* =date t foreign nominal money stock, P_t = date t domestic price level, P_t^* = date t foreign price level, Y_t =date t domestic real income, Y_t^* =date t foreign real income, i_t =date t domestic nominal interest rate, i_t^* =date t foreign interest rate, S_t = current (spot) nominal exchange rate, $\Delta s_{t+1}^e \equiv \left(S_{t+1}^e - S_t\right)/S_t$ = date t expected rate of change in S_t.

Equation (8.1) is the equilibrium condition for the domestic money market. It states that in equilibrium the supply of real money balances (M_t/P_t) equals the demand for real money balances ($Y_t^{\alpha_1} e^{-\alpha_2 i_t}$), where α_1, $\alpha_2 > 0$ are constant parameters measuring the responsiveness of real money holdings to income and the interest rate, respectively. Equation (8.2) expresses the purchasing power parity condition (see, Eq. (6.2)). Equation (8.3) is the uncovered interest parity condition which follows from the assumption of perfect capital mobility and perfect substitutability between domestic and foreign bonds. Equation (8.4) is the equilibrium condition for the foreign money market.

If we take logs of equations (8.1)-(8.4), we can turn the model into a system of linear equations obtaining:

$$
\begin{aligned}
m_t - p_t &= \alpha_1 y_t - \alpha_2 i_t && (8.1a)\\
p_t &= s_t + p_t^* && (8.2a)\\
i_t &= i_t^* + \Delta s_{t+1}^e && (8.3a)\\
m_t^* - p_t^* &= \alpha_1 y_t^* - \alpha_2 i_t^*, && (8.4a)
\end{aligned}
$$

where the lower case letter now denotes logs of the original variables (i.e., $x_t \equiv \ln X_t$ for the generic individual variable X_t).

Solving (8.1a) and (8.4a) for the domestic and foreign price level, yields

$$
\begin{aligned}
p_t &= m_t - \alpha_1 y_t + \alpha_2 i_t && (8.5)\\
p_t^* &= m_t^* - \alpha_1 y_t^* + \alpha_2 i_t^*. && (8.6)
\end{aligned}
$$

Next, substituting (8.5) and (8.6) into (8.2a) and solving for s_t, leads to

$$s_t = (m_t - m_t^*) - \alpha_1 (y_t - y_t^*) + \alpha_2 (i_t - i_t^*), \qquad (8.7)$$

which is known as the 'reduced form' equation for the exchange rate. Equation (8.7) shows that the (spot) exchange rate depends on the relative levels of money supplies $(m_t - m_t^*)$, of national incomes $(y_t - y_t^*)$, and of interest rates $(i_t - i_t^*)$. It predicts that - ceteris paribus - an increase in the domestic money stock relative to the foreign stock leads to a depreciation in the exchange rate ($s_t \uparrow$); an increase in the domestic national income relative to the foreign income causes an appreciation ($s_t \downarrow$); and a rise in the domestic interest rate relative to the foreign rate brings about a depreciation ($s_t \uparrow$).

The rational behind these effects is as follows. Consider first the case of a 5 per cent increase in the domestic money supply while holding all other variables unchanged. By equation (8.5), this leads to a 5 per cent increase in the domestic price, and because PPP holds continuously this will also lead to a 5 per cent increase in the exchange rate. Alternatively, if we take the case of an increase in national income, then money demand will increase; given the money stock, the increased demand for real money balances can only happen if the domestic price level comes down, and this require an appreciation of the domestic currency to maintain purchasing power parity and money market equilibrium. Finally, a rise in the domestic interest rate causes a fall in the demand for money, which in turn require a rise in the domestic price level to maintain money market equilibrium. The rise in the price level then requires a depreciation of the exchange rate to preserve the purchasing power parity condition.

Equation (8.7) can be rewritten so as to include exchange-rate expectations instead of interest-rate differential in the explanatory variables. To see, rewrite (8.3a) as

$$\Delta s_{t+1}^{e} = \Delta s_{t+1} = i_t - i_t^*, \tag{8.8}$$

where $\Delta s_{t+1}^{e} = \Delta s_{t+1}$ under of perfect foresight. Substitute (8.8) into (8.7) for $(i_t - i_t^*)$, to get

$$s_t = (m_t - m_t^*) - \alpha_1 (y_t - y_t^*) + \alpha_2 \Delta s_{t+1}, \tag{8.9}$$

whence, rearranging terms and recalling that $\Delta s_{t+1} \equiv (s_{t+1} - s_t)$,

$$s_t = \frac{1}{1+\alpha_2} \left[(m_t - m_t^*) - \alpha_1 (y_t - y_t^*) + \alpha_2 s_{t+1} \right]. \tag{8.10}$$

Equation (8.10) gives the dynamics of the nominal exchange rate consistent with money market equilibrium. It shows that the dynamics of the nominal exchange rate is driven not only by the 'fundamentals' $[(m_t - m_t^*) - \alpha_1 (y_t - y_t^*)]$ but also by expectations about the future value of the exchange rate s_{t+1}.

Equation (8.10) is a first-order linear difference equation describing the evolution of s_t over time when time changes in discrete steps (i.e., $t = 0, 1, 2, \cdots$). Its solution is obtained by iterating (8.10) forward (see Appendix A at the end of this chapter), to yield

$$s_t = \left(\frac{1+\alpha_2}{\alpha_2} \right)^t (s_0 - F_0) + F_t, \tag{8.11}$$

where s_0 is the initial exchange rate, that is the value of the (spot) exchange rate at $t = 0$,

$$F_0 = \left(\frac{1}{1+\alpha_2} \right) \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2} \right)^i \left[(m_i - m_i^*) - \alpha_1 (y_i - y_i^*) \right]$$

and

$$F_t = \left(\frac{1}{1+\alpha_2} \right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2} \right)^{j-t} \left[(m_j - m_j^*) - \alpha_1 \left(y_j - y_j^* \right) \right].$$

Equation (8.11) is what is known as the forward-looking solution to (8.10). It is made up of two terms: F_t, which is called the *fundamental* or *equilibrium solution*, and $\left[(1+\alpha_2)/\alpha_2 \right]^t (s_0 - F_0)$, which is called the *bubble* component capturing possible deviations from F_t unrelated to the underlying path of the exogenous variables or "fundamentals" of the economy, that is paths where the exchange rate explodes only because it is expected to do so (See Sargent and Wallace, 1973; Flood and Garber, 1980). Equation (8.11) suggests that the nominal exchange rate goes to plus or minus infinity as $t \longrightarrow \infty$ unless the term in brackets $(s_0 - F_0)$ is zero.[1] Therefore, in order to rule out such explosive paths for s_t, we need to impose the restriction

$$s_0 = F_0 = \left(\frac{1}{1+\alpha_2}\right) \sum_{i=0}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^i [(m_i - m_i^*) - \alpha_1 (y_i - y_i^*)] \, .$$

Substituting for s_0 into (8.11) yields

$$s_t = \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} \left[\left(m_j - m_j^*\right) - \alpha_1 \left(y_j - y_j^*\right)\right] , \quad (8.12)$$

which is the *equilibrium* solution or the *saddle path* solution of the exchange rate. Solution (8.12) excluding bubble paths implies that the economy is always on its saddle path and characterizes the equilibrium dynamics of s_t as saddle-point stable.[2]

Equation (8.10) rises two critical issues. The first is to understand why we need a forward solution. The second relates to the economic intuition behind its solution. Let us start with the first issue.

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[1] This because $(1+\alpha_2)/\alpha_2 > 1$, so that

$$\left(\frac{1+\alpha_2}{\alpha_2}\right)^t (s_0 - F_0) \longrightarrow \pm \, \infty \text{ as } t \longrightarrow \infty$$

unless $(s_0 - F_0) = 0$.

[2] See, e.g., Piersanti (2012, Chap. 1).

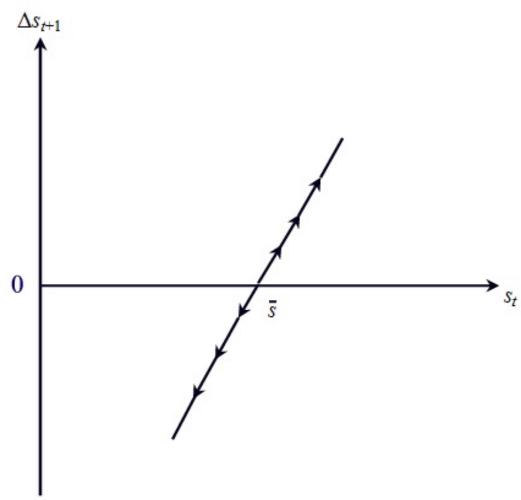

Figure 8.1 The instability of exchange rate.

The key point is that equation (8.9) gives a dynamically unstable exchange rate, as illustrated in the phase diagram in Fig. 8.1. The phase line is upward sloping with a slope given by $(1/\alpha_2) > 0$, which means that the exchange rate has a positive feedback on itself.[3] The higher the level of s_t, the higher its rate of change Δs_{t+1}. There is a unique stationary point with $\Delta s_{t+1} = 0$ shown as $\bar{s}$.[4] To the right of $\bar{s}$, $s_t > \bar{s}$, $\Delta s_{t+1} > 0$ and the exchange rate depreciates forever. To

[3] To check rearrange equation (8.9) to obtain

$$\Delta s_{t+1} = \frac{1}{\alpha_2} s_t - \frac{1}{\alpha_2} \left[(m_t - m_t^*) - \alpha_1 (\bar{y} - \bar{y}^*) \right],$$

whence, differentiating with respect to s_t,

$$\frac{d\Delta s_{t+1}}{ds_t} = \frac{1}{\alpha_2} > 0.$$

[4] To compute the state-state equilibrium point $\bar{s}$ set $\Delta s_{t+1} = 0$ in (8.9) to yield:

$$\bar{s} = \left[(\bar{m} - \bar{m}^*) - \alpha_1 (\bar{y} - \bar{y}^*) \right],$$

where barred variables denote steady-state values.

the left of $\bar{s}$, $s_t < \bar{s}$, $\Delta s_{t+1} < 0$ and the exchange rate continues to appreciate without end. Hence, unless the economy initially jumps at $\bar{s}$, there is no convergence to the equilibrium point $\bar{s}$.

To understand, set for simplicity $y_t = y_t^*$, so that (8.9) reduces to

$$s_t = (m_t - m_t^*) + \alpha_2 \Delta s_{t+1}.$$

Suppose now that the economy is initially in steady state (i.e., in $\bar{s}$), $(m_t - m_t^*)$ is constant and that there is a small rise in s from $\bar{s}$. The rise in the exchange rate means a higher domestic price level and a lower real quantity of money (see Eqs. (8.1a) and (8.2a)), which is consistent with equilibrium only if the demand for money decreases. This requires an increase in the domestic interest rate, which could come about only if agents expect a higher rate of depreciation, given the interest parity condition. As the expected (and actual) rate of depreciation goes up, the inflation rate starts to rise and the real stock of money starts to fall further. Money market equilibrium therefore requires a further increase in the interest rate, which in turn implies a further increase in the depreciation rate. The exchange rate continues to depreciate (at an accelerating speed since Δs_{t+1} increases), the inflation rate and the nominal interest rate increase steadily, and the real stock of money tends to zero as time goes towards infinity. The same unstable path works in reverse for a small drop in s_t from $\bar{s}$.

The monetary approach rules out explosive paths by assuming that the exchange rate jumps initially to $\bar{s}$ and then stays there forever, unless the system is disturbed in some way. This means setting $(s_0 - F_0) = 0$ in equation (8.11) and choose the only solution that places the economy on its saddle-path equilibrium. The non-explosive solution to equation (8.11), that is, equation (8.12), states that the current value of the nominal exchange rate depends on the expected time path of all the variables (or fundamentals) included in the model and is said to be the forward looking solution of s_t. It is the only solution that allows s_t to move discontinuously at each point in time and be thus free to jump at any time.[5]

Solution (8.12) makes the second issue straightforward. It computes the current (spot) exchange rate as the present discounted value of all current and future values of money supply m, of output y, and of exogenous foreign variables m^* and y^*. It implies a unique stationary value for the nominal exchange rate only when all 'fundamentals' (m_t, y_t, m_t^*, y_t^*) are expected to remain constant over the entire future time path. Changes in the fundamentals will alter the time path of s_t,

[5] In the theoretical literature on the solution methods for equations like (8.10), variables whose dynamics follow from the future and are thus free to move discontinuously at each point in time are called jump variables. See, e.g., Blanchard and Kahn (1980), Blanchard (1985), Piersanti (2012, Mathematical Appendix).

setting out an exogenous dynamics that comes from the future through agents' forward expectations.

To illustrate let us focus on the change in fundamentals caused by an increase in the domestic money supply m_t, assuming that all other variables remain fixed. This means that the vector of fundamentals (m_t, y_t, m_t^*, y_t^*) reduces to m_t, and (8.12) can be read as

$$s_t = \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_j. \tag{8.13}$$

Denote the initial expected time path for the money supply as

$$m_j = m_0, \ \forall j \geq t,$$

where m_0 is a constant. From (8.13), the solution for the exchange rate is

$$s_t^0 = \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 = m_0, \tag{8.13}$$

where $s_t^0 = m_0$ is the value of the nominal exchange rate consistent with an expected money supply of m_0.

Suppose that after an unanticipated and once-and-for-all increase in the money supply that leaves the growth rate unaltered (i.e., $\Delta m_t = 0$), the new time path for the money supply is expected to be

$$m_j = m_0 + \zeta, \ \forall j \geq t, \ \zeta > 0.$$

From (8.13), the solution for the nominal exchange rate is

$$s_t = \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} (m_0 + \zeta) \Longrightarrow$$

$$s_t = \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 + \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} \zeta \Longrightarrow$$

$$s_t = s_t^0 + \zeta = m_0 + \zeta, \tag{8.14}$$

so the exchange rate jumps by the same factor ζ as does the money supply. By equation (8.2a), the domestic price level p_t also jumps by the same factor ζ. As a result, the real money stock and all the real variables in the economy do not change, and the jump in the money supply causes only adjustments in s_t and p_t so as to maintain the economy at its steady-state equilibrium position $\bar{s}$. The path of the nominal money stock, the exchange rate, the price level and real money balances are described in figure 8.2, where $m_1 - m_0 = s_1 - s_0 = p_1 - p_0 = \zeta$.

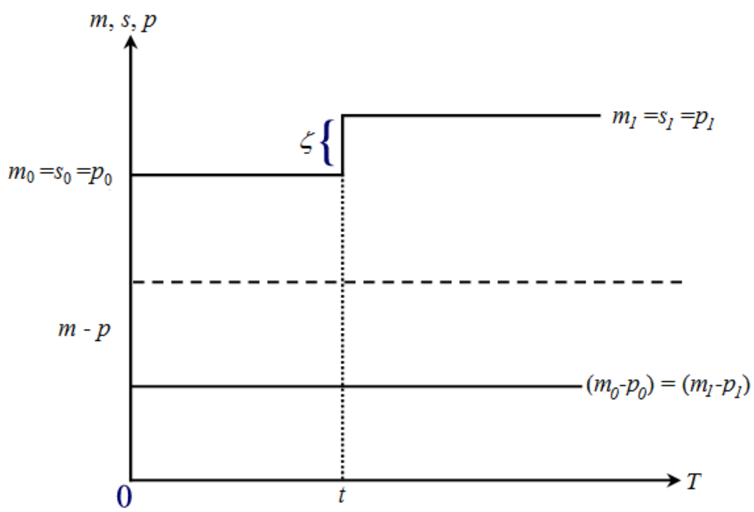

Figure 8.2 Effects of unanticipated monetary expansion.

The economic intuition of this result is simple. Forward-looking agents know that the initial increase in the money supply will be permanent, and with prices fully free-flexible and reflecting all the available information, the exchange rate and the price level must fully respond to the change in m.

Now consider the response of the system to an announcement at time $t = t_0$ that the money stock will increase by ζ at time t_n in the future. The anticipated time profile for m_t is

$$m_j = m_0, \quad \forall j \leq t_n \tag{8.15a}$$

$$m_j = m_0 + \zeta, \quad \forall j \geq t_n. \tag{8.15b}$$

Substituting (8.15) into (8.13), we find that the solution for the nominal exchange rate is

$$s_t = \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{t_n}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t_n}^{\infty}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t}(m_0+\zeta) \Longrightarrow$$

$$s_t = \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{\infty}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t_n}^{\infty}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t}\zeta \Longrightarrow$$

$$s_t = m_0 + \left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n - t} \zeta, \quad \forall t_0 \le t \le t_n, \tag{8.16a}$$

and

$$s_t = \left(\frac{1}{1+\alpha_2}\right) \sum_{j=t}^{\infty} \left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} (m_0 + \zeta) = m_0 + \zeta, \ \ \forall t \ge t_n \tag{8.16b}$$

The time path for the nominal exchange rate is drawn in Fig. 8.3. It shows that s_t jumps by $[\alpha_2/(1+\alpha_2)]^{t_n-t}\zeta$ at time $t = t_0$ and then rises consecutively until time t_n, when the new steady-state value is achieved. No jump occurs at $t = t_n$, when the increase in the money stock takes place. The announcement of the future expansion in the money supply causes the exchange rate (and the domestic price) level to jump immediately at time t_0. This is because individuals anticipate the future depreciation in the exchange rate at $t = t_n$. The increase in s by the amount $[\alpha_2/(1+\alpha_2)]^{t_n-t}\zeta$ at $t = t_0$ therefore discounts to the present the effects of the expected future monetary expansion. The more distant in the future is the expansion, the smaller the current response in the exchange rate. The exchange rate depreciates at an accelerating speed between t_0 and t_n, causing the domestic interest rate to rise in order to preserve equilibrium in the money market where the real money supply is reduced due to the jump in the price level. At time t_n, when the monetary expansion takes place, the rate of depreciation falls back to zero, the domestic interest rate equals the foreign rate i^*, and the real money stock is brought back to its original level.

The time path for s_t is continuous at time $t = t_n$, hence the nominal exchange rate does not jump when m_t changes but at time $t = t_0$, when the news of the future expansion first arrives. However, the expected rate of depreciation does jump at t_n. From (8.16a) and (8.16b), we can see that[6]

$$\Delta s_t = \left(\frac{1}{1+\alpha_2}\right)\left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n-t} \zeta, \quad \forall t_0 \le t \le t_n,$$

[6]To prove, simply observe that the per-period rate of depreciation Δs_t is

$$\Delta s_t = s_t - s_{t-1} = m_0 + \zeta - (m_0 + \zeta) = 0, \quad \forall t > t_n,$$

and

$$\begin{aligned}
\Delta s_t &= m_0 + \left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n-t} \zeta - m_0 - \left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n - t_{-1}} \zeta \Longrightarrow \\
\Delta s_t &= \left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n-t} \zeta \left(1 - \frac{\alpha_2}{1+\alpha_2}\right) = \left(\frac{1}{1+\alpha_2}\right)\left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n-t} \zeta, \quad \forall t_0 \le t \le t_n.
\end{aligned}$$

and

$$\Delta s_t = 0, \quad \forall t \geq t_n.$$

At $t = t_n$ the current (left-hand time) rate of change of currency depreciation exceeds the expected (right-hand time) depreciation rate by $[1/(1+\alpha_2)]\,\zeta > 0$. The fall of Δs_t to zero at time t_n induces a fall in the home interest rate, that increases the demand for money to compensate for the jump in the money supply at time t_n. Money market equilibrium is maintained and there is no need for the exchange rate to jump.

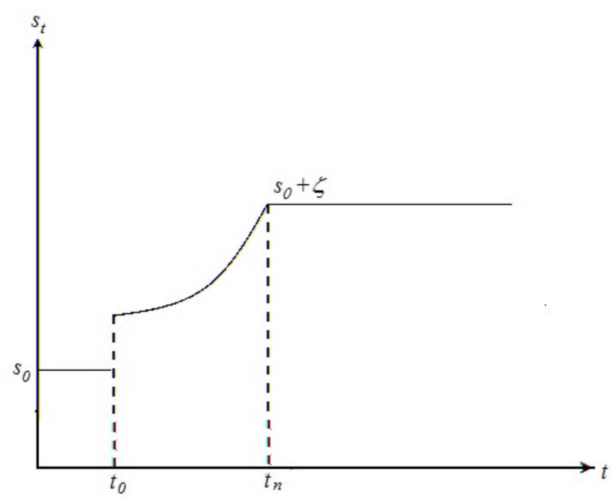

Figure 8.3 Effects of anticipated monetary expansion.

Consider now the response of the system to a jump in the money stock that is known to be temporary. The time profile of the money supply is now

$$\begin{aligned} m_j &= m_0 + \zeta, \quad \forall j \leq t_n \\ m_j &= m_0, \quad \forall j \geq t_n, \end{aligned}$$

and the solution for s_t is

$$s_t = \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{t_n}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t}(m_0+\zeta) + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t_n}^{\infty}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 \Longrightarrow$$

$$s_t = \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{\infty}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{t_n}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} \zeta \Longrightarrow$$

$$s_t = m_0 + \zeta\left[1 - \left(\frac{\alpha_2}{1+\alpha_2}\right)^{t_n - t}\right], \quad \forall t_0 \le t \le t_n, \tag{8.17a}$$

$$s_t = \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{\infty}\left(\frac{\alpha_2}{1+\alpha_2}\right)^{j-t} m_0 = m_{0,} \;\; \forall t \ge t_n. \tag{8.17b}$$

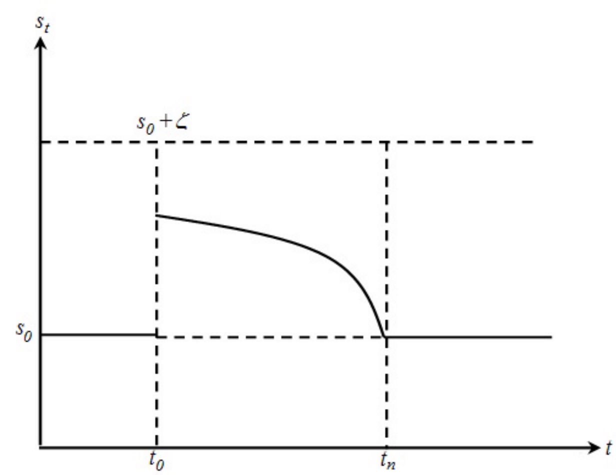

Figure 8.4 Effects of a temporary monetary expansion.

Figure 8.4 displays the time path of the nominal exchange rate and shows the two phases of adjustment. At $t = t_0$, s_t jumps upward, but less than it would have done if the increase in m_t had been permanent. The exchange rate falls (i.e., appreciates) until time $t = t_n$, when the change in the money stock is reversed and the initial equilibrium level of s_t is restored.

The intuition underlying this result is the following. Forward-looking agents know that at time $t = t_1$ both the exchange rate and the real money balances will be back at their initial equilibrium levels.

Before the increase in m is reversed, the real money stock is higher and the real demand for money must be higher. This requires a lower interest rate, which is possible only if the exchange rate is expected to appreciate. Thus, immediately before t_n, s_t must be falling and above its steady-state equilibrium value. Going backward, the exchange rate must first jump upwards at t_0 and then appreciate gradually towards its old level s_0.

Similar results can be obtained if we consider exogenous shocks where the growth rate of the money supply and not simply the level is modified.[7]

8.3 Sticky Prices and Exchange-Rate Overshooting: the Dornbusch Model

The monetary model discussed earlier assumes that the PPP condition holds continuously, and that prices, wages and the exchange rates are fully free flexible. Under these assumptions, the model predicts that the exchange rate, and prices via the PPP condition, must instantly adjust to accommodate monetary disturbances and preserve long-run stability. As such, the model is of no use in explaining the high levels of exchange rate volatility and prolonged departures from PPP observed in the real world.

By relaxing the assumptions about PPP and price flexibility, Dornbusch (1976) proposed a model of exchange rate volatility now considered to be a classic in the field. The model builds around the following key hypotheses:

- Prices and wages are sticky, or slow to change in the short run.
- Expectations of exchange rate changes are perfect, or rational.
- Capital mobility is perfect and all securities are perfect substitutes, so that the UIP condition holds at all time.
- The home country is a 'small open economy', which means that all foreign variables are exogenously fixed.

[7] Analytical details on a number of possible cases including changes in the growth rate of m_t, can be found. e.g., in Piersanti (2012, Chap. 1, Sect. 1.4, Permanent and Temporary Changes in the Exogenous Variables and Exchange Rate Dynamics, pp. 22-27).

The equations of the model are as follows

$$Y_t^d = \bar{Y}\left(\frac{S_t P^*}{P_t}\right)^{\delta}\left(\frac{1+i_t}{\frac{P_{t+1}^e - P_t}{P_t}}\right)^{-\gamma} G_t \quad (8.18)$$

$$\frac{P_{t+1}-P_t}{P_t} = \left(\frac{Y_t^d}{\bar{Y}}\right)^{\theta} \quad (8.19)$$

$$\frac{M_t}{P_t} = Y_t^{\phi}(1+i_t)^{-\eta} \quad (8.20)$$

$$(1+i_t) = (1+i^*)\frac{S_{t+1}^e}{S_t}, \quad (8.21)$$

where Y_t^d is aggregate demand for home-country output, $\bar{Y}$ is the "natural" rate of output, and the other variables have the same meaning as before. Equation (8.18) states that aggregate demand is positively linked to the real exchange rate $(S_t P^*/P_t)$ and to government spending G, and negatively linked to the real interest rate $\left[(1+i_t)/\left(\left(P_{t+1}^e - P_t\right)/P_t\right)\right]$. Equation (8.19) states that prices move in response to excess demand in the goods market. Equation (8.21) is the condition for asset market equilibrium. Equation (8.21) is the UIP condition. Taking logs, we can linearize the model to obtain

$$y_t^d = \bar{y} + \delta\left(s_t + p^* - p_t\right) - \gamma\left[i_t - (p_{t+1}-p_t)\right] + g_t \quad (8.18a)$$

$$p_{t+1} - p_t = \theta\left(y_t^d - \bar{y}\right) \quad (8.19a)$$

$$m_t - p_t = \phi y_t - \eta i_t \quad (8.20a)$$

$$i_t = i^* + (s_{t+1} - s_t), \quad (8.21a)$$

where lower-case letters are logs of the corresponding upper-case letter variables, and $p_{t+1}^e = p_{t+1}$ and $s_{t+1}^e = s_t$ given perfect-foresight.

To solve the model and analyze the behavior of the exchange rate following monetary and fiscal shocks, substitute (8.18a) into (8.19a) and (8.21a) into (8.20a), to yield

$$p_{t+1} - p_t = \theta\left\{\delta\left(s_t + p^* - p_t\right) - \gamma\left[i_t - (p_{t+1}-p_t)\right] + g_t\right\} \quad (8.22)$$

$$m_t - p_t = \phi y_t - \eta\left[i^* + (s_{t+1}-s_t)\right], \quad (8.23)$$

whence, using (8.20a) to substitute for i_t into (8.22) and solving (8.23) for $(s_{t+1} - s_t)$,

$$\Delta p_t \equiv p_{t+1} - p_t = \frac{\theta}{1-\theta\gamma}\left[\delta\left(s_t + p^* - p_t\right) - \frac{\gamma}{\eta}\left(\phi y_t - m_t + p_t\right) + g_t\right] \quad (8.22a)$$

$$\Delta s_t \equiv s_{t+1} - s_t = \frac{1}{\eta}\left(\phi y_t - m_t + p_t\right) - i^*, \quad (8.23a)$$

so obtaining a dynamical system of two first-order, linear difference equations governing price changes $(p_{t+1} - p_t)$ and exchange rate movements $(s_{t+1} - s_t)$ in this economy.

Let us now solve the model for the long-run, or steady-state equilibrium. This is computed by setting $\Delta p_t = \Delta s_t = 0$, to yield

$$p_{t+1} = p_t = \bar{p},\ s_{t+1} = s_t = \bar{s},\ y_t^d = \bar{y},\ i_t = i^*,\ \forall t, \tag{8.24}$$

from (8.19a) and (8.21a), and

$$\bar{s} = (\bar{p} - p^*) + \frac{1}{\delta}(\gamma i^* - \bar{g}) \tag{8.25}$$

$$\bar{p} = \bar{m} + \eta i^* - \phi \bar{y}, \tag{8.26}$$

from (8.22) and (8.23).

Equations (8.24)-(8.26) shows that in steady-state equilibrium, all variables take on constant values. Hence, $\Delta p_t = \Delta s_t = 0$, the home-country output y_t^d equals the full-employment equilibrium level $\bar{y}$, the domestic interest rate i_t equals the foreign rate i^*, and the steady-state exchange rate and price level are proportional to the relative price levels $(\bar{p} - p^*)$ and to the (exogenously) given money supply $\bar{m}$. Notice, however, that there are additional terms. From (8.19a), δ is the real exchange-rate elasticity of domestic aggregate demand. If home and foreign goods were perfect substitute, then $\delta \to \infty$ and (8.25) would collapse to $\bar{s} = \bar{p} - p^*$, which would be (the long run) PPP condition. The variables i^* and $\bar{y}$ in (8.26) aim to capture, instead, the additional effects of the foreign interest rate and of "potential", or natural rate of output on the long-run equilibrium level of domestic price.

If we now subtract (8.25) and (8.26) from (8.22a) and (8.23a) and let m_t, g_t and y_t be exogenously fixed at $\bar{m}, \bar{g}$ and $\bar{y}$, we can express price and exchange rate dynamics in terms of deviations from their long-run equilibrium values $\bar{p}$ and $\bar{s}$, namely

$$p_{t+1} - \bar{p} = \left[1 - \frac{\theta(\delta + \gamma/\eta)}{1 - \theta\gamma}\right](p_t - \bar{p}) + \frac{\theta\delta}{1 - \theta\gamma}(s_t - \bar{s}) \tag{8.27}$$

$$s_{t+1} - \bar{s} = \frac{1}{\eta}(p_t - \bar{p}) + (s_t - \bar{s}). \tag{8.28}$$

Equations (8.27) and (8.28) summarize the dynamics of the model and identify the path, or solution trajectories that drive the economy towards the long-run equilibrium point, or stationary state $(\bar{p}, \bar{s})$. The analytical solution of the model is rather complex and is given in Appendix B. Here, we use a graphical representation known as phase diagram. This is accomplished by plotting the evolution of p_t and s_t in the (s, p) plane as shown in Fig. 8.5. The figure displays (i) two (demarcation) curves defining the steady-state relationships where

$\Delta p_t = \Delta s_t = 0$, (ii) the paths of variables p_t and s_t away from the steady state. Let us focus on the $\Delta p_t = 0$ and $\Delta s_t = 0$ schedules first.

To obtain the $\Delta p_t = 0$ schedule, add and subtract p_t from the left-hand side of (8.27), to yield

$$\Delta p_t \equiv p_{t+1} - p_t = -\frac{\theta\left(\delta + \gamma/\eta\right)}{1-\theta\gamma}\left(p_t - \bar{p}\right) + \frac{\theta\delta}{1-\theta\gamma}\left(s_t - \bar{s}\right),$$

whence, setting $\Delta p_t = 0$,

$$p_t - \bar{p} = \left(\frac{\eta\delta}{\eta\delta + \gamma}\right)\left(s_t - \bar{s}\right), \tag{8.29}$$

which is the straight line crossing the steady-state point $(\bar{s}, \bar{p})$ with a slope $[\eta\delta/\left(\eta\delta + \gamma\right)]$ in figure 8.5.

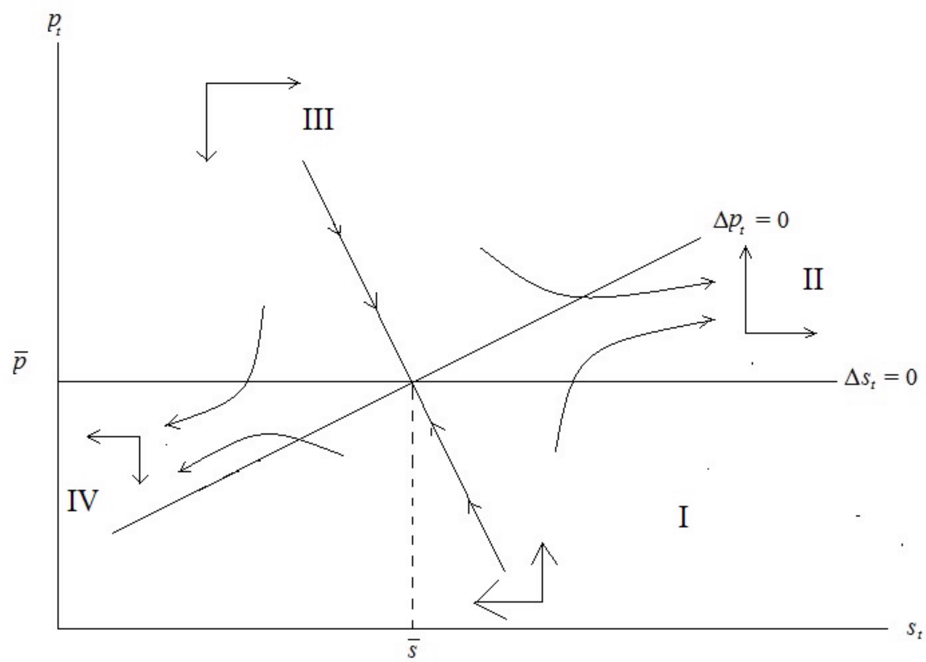

Figure 8.5 Equilibrium and dynamics in the Dornbusch model.

Follow the same steps to get from (8.28) the $\Delta s_t = 0$ schedule

$$p_t = \bar{p}, \tag{8.30}$$

which is the straight line starting at $\bar{p}$ and crossing the steady-state point $(\bar{s}, \bar{p})$ with a slope equal to zero.

Equation (8.29) shows that the $\Delta p_t = 0$ schedule is upward-sloping and has a slope lesser the one, that is

$$\frac{\eta\delta}{\eta\delta + \gamma} < 1.$$

It also implies that aggregate demand equals aggregate supply so that price inflation is zero. The rationale behind the positive relationship between p_t and s_t along the goods market equilibrium curve is as follows. A depreciation of the exchange rate ($s_t \uparrow$) leads to an excess demand for domestic goods as exports increase. To restore equilibrium, price must rise so as to offset the boost on demand of a higher exchange rate.

Equation (8.30) shows that the $\Delta s_t = 0$ schedule along which we have money market equilibrium is flat: there is only one price level $\bar{p}$, at which the home interest rate equals the foreign rate and hence expected and actual depreciation is zero.

The point where Δp_t and Δs_t are simultaneously equal to zero (i.e., $\Delta p_t = \Delta s_t = 0$) is the steady-state or long-run equilibrium of the model.

Equations (8.29) and (8.30) divide the (s, p) plane in four regions showing the dynamics of p_t and s_t away from the steady state. To see, take equation (8.28) and rewrite it as

$$\Delta s_t \equiv s_{t+1} - s_t = \frac{1}{\eta}\left(p_t - \bar{p}\right),$$

which immediately show that $\Delta s_t \gtrless 0$ according to whether $p_t \gtrless \bar{p}$. Hence, starting from the line where $\Delta s_t = 0$, a rise in p will drive us in the region in which $\Delta s_t > 0$, whereas a decline in p will move us in the region where $\Delta s_t < 0$. This is why the exchange rate increase (decrease) over time above (below) the line of asset market equilibrium, and the arrows of motion along the s_t axis point to the right (left) in figure 8.5.

Similarly, focusing on (8.19a) we find that $\Delta p_t \gtrless 0$ according to whether $y_t^d \gtrless \bar{y}$. Hence, starting from the line where $\Delta p_t = 0$, a rise in s will put us in the region in which $\Delta p_t > 0$, as the rise of the exchange rate pushes aggregate demand above the aggregate supply, whereas a decline in s will move us in the region where $\Delta p_t < 0$, as y_t^d is driven below $\bar{y}$.This means that p_t is rising (falling) below (above) the line of goods market equilibrium, and the arrows of motion point toward the $\Delta p_t = 0$ line. Combining these movement we obtain the phase diagram shown in figure 8.5.

The figure reveals that there exits only one path, called the saddle path (see, Appendix B), along which the system converges to the steady-state equilibrium point. This is the path, or trajectory crossing regions III and I in Fig. 8.5. Any point outside this trajectory will move further and further away from the steady-state point $(\bar{s}, \bar{p})$. Thus, unless the economy jumps initially to the saddle path following a monetary or fiscal disturbance, it will never attain the equilibrium point.

Let us now use the model to analyze the effects of different monetary and fiscal disturbances on price level and exchange rates. Imagine the economy is initially at the long-run equilibrium point $E_0 = (\bar{s}_0, \bar{p}_0)$ corresponding to values $\bar{m}_0$ and $\bar{g}_0$ of the money stock and government expenditure. Suppose now there is an unanticipated, and permanent increase in the money stock, from $\bar{m}_0$ to $\bar{m}_1 = \bar{m}_0 + \zeta,\ \zeta > 0$. From (8.25) and (8.26), we find that the new steady-state values of the price level and the exchange rate are

$$\bar{p}_1 - \bar{p}_0 = \bar{m}_1 - \bar{m}_0 = \zeta = \bar{s}_1 - \bar{s}_0, \tag{8.31}$$

so that in the long run p_t and s_t will increase by the same factor ζ as does the money supply, leaving real variables unaffected.[8] This long-run effect of a monetary expansion is shown in Fig. 8.6 as a shift from the initial equilibrium point E_0 to new stationary state $E_1 = (\bar{s}_1, \bar{p}_1)$ along the straight line $(0R)$ with slope equal to one.

[8] To check, observe from equations (8.25) and (8.26) that

$$\bar{p}_0 = \bar{m}_0 + \eta i^* - \phi\bar{y}; \quad \bar{s}_0 = (\bar{p}_0 - p^*) + \frac{1}{\delta}(\gamma i^* - \bar{g}_0)$$

and

$$\bar{p}_1 = \bar{m}_1 + \eta i^* - \phi\bar{y}; \quad \bar{s}_1 = (\bar{p}_1 - p^*) + \frac{1}{\delta}(\gamma i^* - \bar{g}_0).$$

Hence,

$$\bar{p}_1 - \bar{p}_0 = (\bar{m}_1 + \eta i^* - \phi\bar{y}) - (\bar{m}_0 + \eta i^* - \phi\bar{y}) = \bar{m}_1 - \bar{m}_0,$$

and

$$\bar{s}_1 - \bar{s}_0 = \left[(\bar{p}_1 - p^*) + \frac{1}{\delta}(\gamma i^* - \bar{g}_0)\right] - \left[(\bar{p}_0 - p^*) + \frac{1}{\delta}(\gamma i^* - \bar{g}_0)\right] = \bar{p}_1 - \bar{p}_0.$$

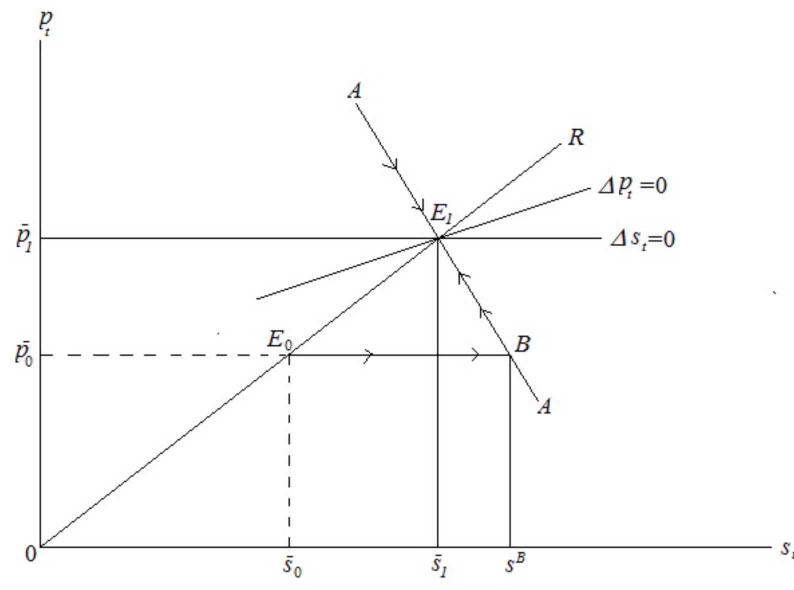

Figure 8.6 Exchange-rate overshooting.

In the short run, however, the economy cannot jump instantaneously to E_1 because prices are sticky and can adjust only gradually to the monetary shock. Hence, the change in m will have real effects, and this means that the full burden of adjustment must initially fall on the exchange rate which is freely flexible. Figure 6 shows the dynamics of this adjustment process. The exchange rate first jumps to B with price stuck at $\bar{p}_0$, so as to place the economy on the saddle path AA leading to the new steady state. Then, the exchange rate appreciates gradually and prices start rising until the new equilibrium point E_1 is reached.

Notice, that the exchange rate exceeds its long-run equilibrium value $\bar{s}_1$ at the start, giving rise to the well known phenomenon of *exchange rate overshooting*. The reason why s_t exceeds the steady-state value $\bar{s}_1$in the short run is as follows. With price assumed to be sticky in the short run, the increase in money supply brings about an increase in the real money supply and hence a fall in the domestic interest rate i_t. Since the UIP condition is assumed to hold continuously, the fall in i_t requires that international investors expect the exchange rate to appreciate in the future to compensate for the lower i_t, otherwise, the rates of return on domestic bonds would be less than foreign bonds, and agents would be less willing to hold domestic currency. Such an appreciation is possible only if the exchange rate rise initially above its long-run level, that is, if s_t jumps from $\bar{s}_0$ to s_B, overshooting the long-run equilibrium value $\bar{s}_1$. Then, the exchange rate appreciates and domestic price rise to drive the economy toward the equilibrium state E_1 along the stable trajectory AA. A number of factors come into play to move s_t and p_t to their long-run equilibrium values. The initial rise in the exchange rate produces an excess demand in the goods markets that is removed gradually over time as domestic price adjust. As domestic price rise, the real money supply reduces and the domestic interest rate increase. This, in turn, leads to a fall in s_t (i.e., the exchange rate appreciates), thus reversing some of the initial rise. This adjustment process stops only when the economy finds itself on the stationary state E_1.

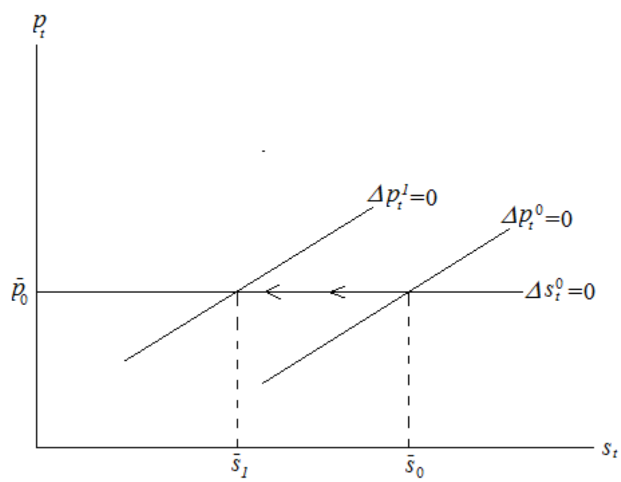

Figure 8.7 Effects of an increase in government spending.

The second policy experiment we analyze is an unexpected, permanent increase in government spending on domestic goods. Suppose there is a one-time (permanent) increase in g, from $\bar{g}_0$ to $\bar{g}_1 = \bar{g}_0 + \xi$, $\xi > 0$. From (8.25) and (8.26), we find that

$$\frac{\partial \bar{s}}{\partial \bar{g}} = -\frac{1}{\delta} < 0, \quad \text{and} \quad \frac{\partial \bar{p}}{\partial \bar{g}} = 0.$$

As a result,

$$\bar{s}_1 - \bar{s}_0 = -\frac{\xi}{\delta}, \quad \text{and} \quad \bar{p}_1 = \bar{p}_0,$$

so that the rise in government spending has no effect on the price level and requires a fall in the exchange rate, which can take place through an appreciation of the home currency that deflects foreign demand away from domestic output. Since the $\Delta p_t = 0$ line also changes, through the change in $\bar{s}$ brought about by the rise in $\bar{g}$, the adjustment to the steady state is immediate as shown in figure 8.7.[9] Hence, there is no overshooting of the exchange rate under fiscal policy shocks.

8.4 The Portfolio Balance Model

The monetary models discussed in Sections 8.2 and 8.3 rest on the crucial assumption that domestic and foreign assets are perfect substitutes. This implies that the expected return on domestic and foreign bonds are always equalized and that a unique global asset market for such identical (perfect substitute) bonds be included in the model. This assumption, however, is hardly met in the real world where imperfect as opposed to perfect substitutability is commonly observed. There are a number of reasons why domestic and foreign assets can be imperfect substitutes: liquidity, tax treatment, default risk, political risk, inflation risk, and exchange rate risk. For simplicity, we can fix all these factors in two main component: currency risks and country risks. Currency risks arise because domestic and foreign assets are denominated in different currencies; country risks arises because assets are issued by countries with different political regimes and legal administrations. Plainly, the existence of such risks means that the UIP conditions no longer holds exactly.

A distinguishing feature of the portfolio balance model is the abandon of the assumption of perfect substitutability between the different type assets. This means that the expected return on domestic and foreign assets no longer have to be equal and that international investors will require a higher expected return on the relative risky asset as compared to the less one. This additional expected return is known as the

[9] Notice, that the leftward shift in the $\Delta p_t = 0$ line in Fig.8.7, follows from the change in the intercept term of equation (8.29),

$$\left(\frac{\eta\delta}{\eta\delta + \gamma}\right)\bar{s},$$

due to the fall in $\bar{s}$.

risk premium and implies that the interest parity condition changes to

$$i_i = i^* + \Delta s^e_{t+1} + \rho,$$

where ρ is the risk premium, or additional return required by investors on the relatively risky assets. The extent of this additional return will affect the allocation of wealth between domestic and foreign bonds.

The model was first developed by Branson (1977, 1981, 1984) and Kouri (1976, 1983), and has been subsequently modified and extended in various directions by Dornbusch and Fischer (1980), Obstfeld (1980), Allen and Kenen (1980), Tobin and de Macedo (1981), Branson and Henderson (1985). In this section, we restrict our discussion to the basic version of the model developed by Branson (1977) and Kouri (1976). The model uses the following simplifying assumption:

- Prices are sticky in the short run.
- Real domestic output is given at the full employment equilibrium level.
- Financial markets includes three assets: domestic money (M), which is non-interest bearing, domestic bonds (B), which are denominated in domestic currency and bears the domestic interest rate, and foreign assets (F), which are denominated in foreign currency and bears the world interest rate.

The model focuses on a small country, which means that the world interest rate is exogenously given and that domestic assets are held solely by resident. Given these assumption we can define total financial wealth (W) as

$$W \equiv M + B + SF, \tag{8.32}$$

and set out the equilibrium conditions in asset markets as follows

$$\begin{aligned} M &= M(i, i^*, W), \quad M_i < 0, M_{i^*} < 0, M_W > 0 && (8.33) \\ B &= B(i, i^*, W), \quad B_i > 0, B_{i^*} < 0, B_W > 0 && (8.34) \\ SF &= F(i, i^*, W), \quad F_i < 0, F_{i^*} > 0, F_W > 0. && (8.35) \end{aligned}$$

Equations (8.33)-(8.35) state that the demand for each asset is a function of its own rate of return, the rate of return on alternative assets, and of total wealth, thus introducing wealth effects in an open economy. Notice also that the balance-sheet identity (8.32) together with the assumption that assets are gross substitute, imply the following adding-up constraints

$$M_i + B_i + F_i = 0 \tag{8.36}$$

and

$$M_W + B_W + F_W = 1. \tag{8.37}$$

Let us now examine the working of the model by assuming that the stock of all three assets are exogenously given and that agents have static expectations (i.e., $(dS_t/dt) = 0$), for simplicity. We begin in full asset market equilibrium as shown in Fig. 8.8. The figure displays three equilibrium functions: the MM curve, the BB curve, and the FF curve. The MM curve shows the different combinations of domestic interest rate and exchange rate for which money supply is equal to money demand. The curve has a positive slope given by

$$\frac{di}{dS} = -\frac{M_W}{M_i}F > 0, \tag{8.38}$$

which results from (8.33) by taking the total differential

$$dM = M_i di + M_{i^*} di^* + M_W(dM + dB + FdS + SdF),$$

setting $dM = dB = dF = 0$, and solving for (di/dS). The reason for the positive relationship between i and S along the MM curve is as follows. A depreciation of the exchange rate rises the domestic currency value of foreign bonds SF. Therefore, total wealth rises increasing money demand. To restore equilibrium in the money market, the domestic interest rate must rise. By contrast, an increase in the money stock shifts the MM curve to the right, since a lower domestic interest rate is required to increase money demand at a given exchange rate.

The BB curve shows the various combinations of domestic interest rate and exchange rate for which the bond market is always in equilibrium. It slopes downward with a slope given by

$$\frac{di}{dS} = -\frac{B_W}{B_i}F < 0. \tag{8.39}$$

This is obtained from (8.34) by taking the total differential

$$dB = B_i di + B_{i^*} di^* + B_W(dM + dB + FdS + SdF),$$

setting $dM = dB = dF = 0$, and solving for (di/dS). The reason for such a negative relationship is simple. A rise in S rises the demand for domestic bonds through the wealth effect. Therefore, to restore equilibrium in the domestic bonds market, the interest rate i must drop. Alternatively, an increase in the supply of domestic bonds shifts the BB curve upward, since a higher domestic interest rate is required to rise the demand of domestic bonds at a given exchange rate.

Finally, the foreign assets equilibrium curve, FF, is downward sloping and has a slope given by

$$\frac{di}{dS} = \frac{(1 - F_W)}{F_i}F < 0. \tag{8.40}$$

This results from (8.35) by taking the total differential

$$d\,(SF) = F_i di + F_{i^*} di^* + F_W(dM + dB + FdS + SdF),$$

setting $dM = dB = dF = 0$, and solving for (di/dS). The reason for this negative relationship is as follows. A rise in the interest rate reduces the demand for foreign assets, and this must be offset by a fall (appreciation) of the exchange rate to restore equilibrium in the foreign bonds market. Given the adding-up constraints, it follows that the FF curve must be steeper than the BB curve in the (S, i) plane. This means that changes in domestic interest rate affect the demand for domestic bonds more than they influence the demand for foreign bonds. An increase in the stock of foreign bonds shifts the FF curve to the left, since a lower domestic interest rate is needed to increase the demand of foreign assets for a given exchange rate.

The full equilibrium in the asset markets is when all three markets - that is, the money market and the domestic and foreign bond markets - clear. This occurs at the interest rate $\bar{i}$ and the exchange rate $\bar{S}$ where the unique global equilibrium of the model is found, as shown in Fig. 8.8.

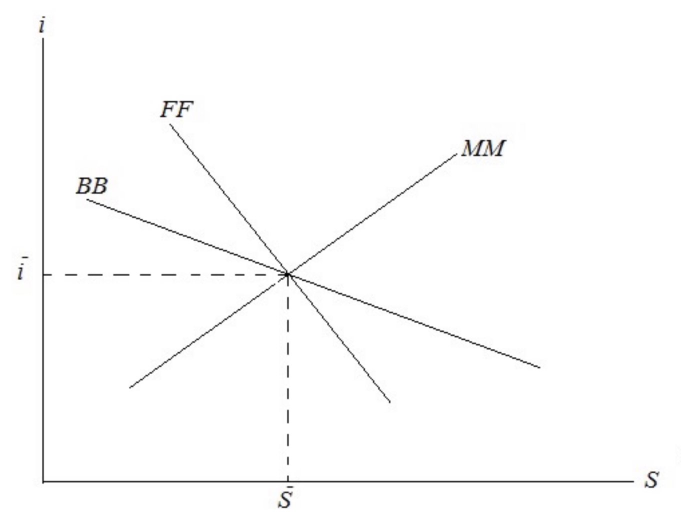

Figure 8.8 Asset market equilibrium.

8.4.1 The Short-Run Equilibrium

We now use the model to discuss the short-run effects of alternative monetary policy strategies involving changes in M, B, and F. We begin by examining the case of so called *open-market operations* (OMO). These are defined as the buying or selling of domestic bonds in the open market for the purpose of regulating the money supply in the economy. Suppose the monetary authority conducts an expansionary OMO by purchasing domestic bonds from the private agents with the newly-increased money stock. This means an increase in money holdings and an equivalent fall in domestic bonds in agents' portfolios; that is $dM = -dB$. These effects are shown in figure 8.9 as a rightward shift of the MM line and a leftward shift of BB line. The net outcome is a rise (depreciation) in the exchange rate and a fall in the domestic interest rate. The reason is straightforward. With an expansionary open market operation total financial wealth is unaltered, as the increase in M is accompanied by a parallel fall in B. The increase in the money stock leads to an excess supply in the money market and to a fall in the domestic interest rate. However, the increase in M also creates an excess supply of money in agents' portfolios which raises the demand for domestic and foreign bonds. This, in turn, brings about a further fall in the domestic interest rate and a depreciation of the home currency, which raises the value of foreign bond holdings and drives asset markets to the new equilibrium point $\left(\bar{i}_1, \bar{S}_1\right)$.

Let us now turn to the case of *foreign-exchange operations* (FXO). These are defined as the buying or selling of foreign bonds by the monetary authority. Thus, if the central bank conducts an expansionary FXO there will be an increase in money holdings and a parallel fall in foreign bonds in agents' portfolio. These effects are shown in figure 8.10 as a downward shift of the money market curve from MM_0 to MM_1, and an upward shift of the foreign bond curve from FF_0 to FF_1, which causes a fall in the domestic interest rate and a depreciation of the exchange rate. The fall in i is required because the expansionary FXO generates an excess supply of money on the money market. The rise in S is required because the FXO causes a shortage of foreign assets in agents' portfolio, which can only be met by a depreciation of the exchange rate which increases the domestic currency value of the remaining holdings of foreign assets. As a result, the final asset market equilibrium is attained at $\left(\bar{i}_1, \bar{S}_1\right)$ with a lower interest rate and a higher exchange rate.

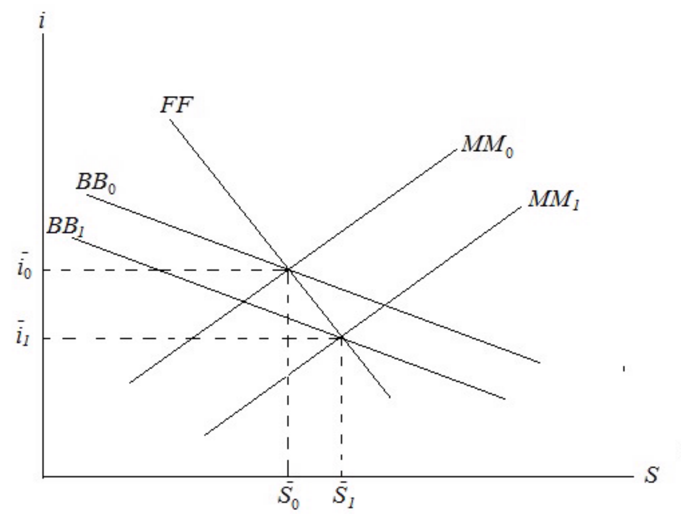

Figure 8.9 The effects of an open market operation.

Observe that while qualitatively similar, the effects on i and S of open-market and foreign-exchange operations differ quantitatively: the OMO causes a larger drop in the domestic interest rate, while the FXO causes a larger depreciation of the exchange rate. This result follows from the fact that an OMO leads to a fall in agents' holdings of domestic bonds, while an FXO does not. Therefore, an OMO gives rise to a greater shortage of domestic bonds which can be met only by a greater contraction in the domestic interest rate; whereas an FXO gives rise to a shortage of foreign bonds which can be met only by a stronger depreciation of the exchange rate.

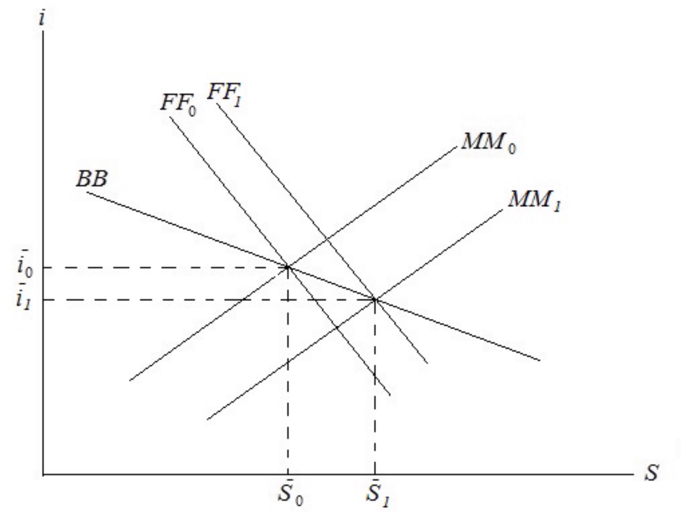

Figure 8.10 The effects of a foreign exchange operations.

Open-market and foreign-exchange operations can be combined to perform so called *sterilized operations* (SO). These refer to an exchange of domestic bonds for foreign bonds so as to leave the money stock unchanged. This means that the central bank firstly purchase foreign assets with domestic money, then offset the increase in M by selling domestic bonds; that is, $dM = -SdF$ and $dB = -dM$, so that $dB = -SdF$. The effects of an SO are shown in Fig. 8.11.

Both the BB and the FF lines shift upward because the SO increases the supply of domestic bonds and reduces agents' holding of foreign assets; whereas the MM curve remains unchanged because the domestic money stock is left unaltered. The net outcome is a rise in the rate of interest from i_0 to i_1 and a depreciation of the exchange rate from S_0 to S_1. The rise in i follows from the excess supply of domestic bonds in agents' portfolio which pushes down the price of domestic bonds. The depreciation of the exchange rate is required to remove the shortage of foreign assets in agents' portfolio caused by the SO. The higher interest rate pushes up the demand for domestic bonds and this helps to restore equilibrium in the bond market; the higher exchange rate raises the domestic currency value of the remaining foreign assets and helps to restore the desired asset holdings. Hence, with sterilized operations the monetary authority alters the composition of bond holdings in agents' portfolio.

Finally, we can illustrate the impact of a pure increase in the domestic money stock in figure 8.12. The MM line shifts downward, since with a larger money stock the domestic interest rate declines and the exchange depreciates to maintain asset market equilibrium. However, the increase in M also increase total wealth and therefore the demand for domestic and foreign bonds, shifting the BB and FF curves to BB_1 and FF_1. As a result, the final equilibrium is at point $\left(\bar{i}_1, \bar{S}_1\right)$ with a further reduction in the domestic interest rate and a lower depreciation of the exchange rate.

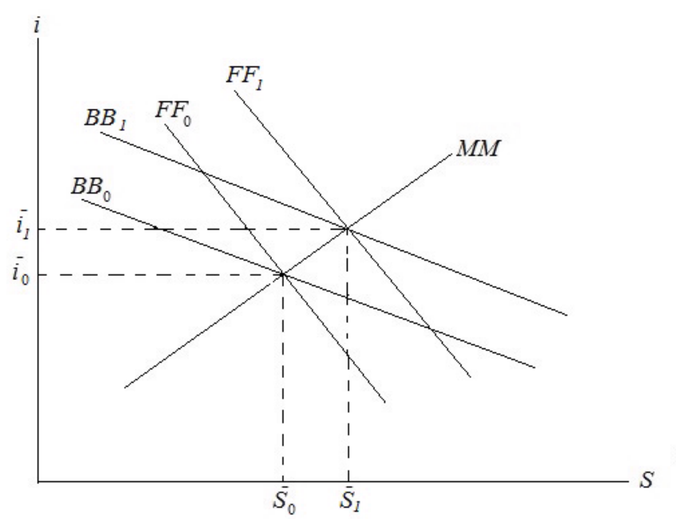

Figure 8.11 The effects of a sterilized operation.

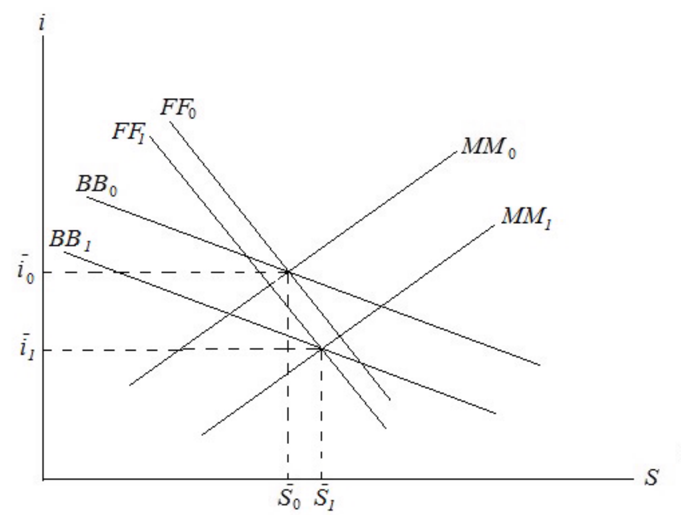

Figure 8.12 The effects of a monetary expansion.

8.4.2 The Long-Run Equilibrium

Asset market equilibrium, however, does not necessarily describe the long-run equilibrium of the economy. To find the long-run equilibrium we need to include the current account into the model. Under the assumptions of a fixed real income, this can be accomplished by adding a current account equation of the form

$$CA = TB(Q) + i^{*}F, \quad CA_{Q} > 0, \tag{8.41}$$

where $Q \equiv (SP^{*}/P)$. The first term relates the trade balance (sum of balance on goods and services) to the real exchange rate Q, and assumes that the Marshall-Lerner conditions are satisfied. The second term refers to the interest income accruing from foreign asset holdings.

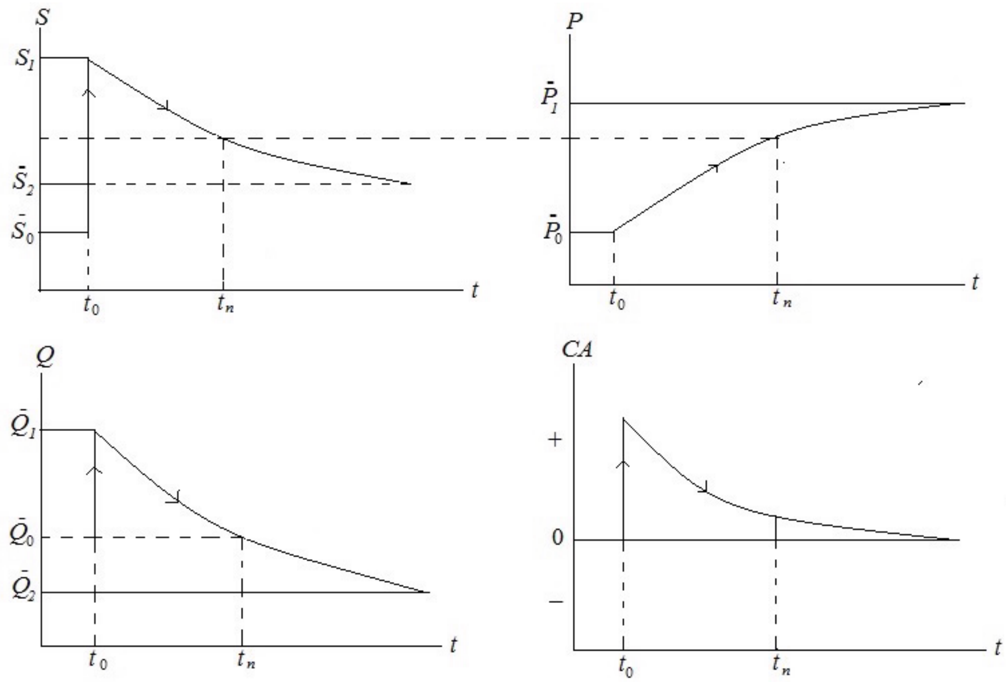

Figure 8.13 The dynamics of the portfolio balance model.

The model shares with Dornbusch (1976) the overshooting in the exchange rate that follows from changes in the money supply; but the key difference is that the emphasis is now on the portfolio effects of the current account response to exchange rate changes. The dynamics of the model is as follows.

Let us assume that the economy is initially at a full equilibrium position, where asset markets clear, and the real exchange rate is such that the trade balance and the current account be equal to zero, that is, $TB = CA = 0$. Suppose there is now an increase in the money stock, say by an OMO, FXO, or a pure M rise. This sets off a dynamic adjustment process in the exchange rate, prices, and the current account that is best described using Fig. 8.13. The money stock expansion causes a rise in the nominal exchange rate from $\bar{S}_0$ to S_1; this, under the sticky price assumption, means that the real exchange rate, Q, also depreciates, turning the trade balance and hence the current account into surplus. As a consequence, agents start accumulating foreign assets, whereas the nominal exchange rate gets going appreciating. As domestic prices move towards $\bar{P}_1$ during the transition from the short-run to the long-run equilibrium, the real exchange rate also appreciates. The nominal and real exchange-rate appreciation, in turn, erodes the country's competitive advantage, so reducing the trade and current account surplus. At time t_n, the increase in S has been just proportional to the price rise and the real exchange rate, Q, is again back to its original value $\bar{Q}_0$. Nevertheless, this is not the final equilibrium position yet, as the accumulation of foreign assets that follows from the surplus in the current account now results in an increase in the interest income from the holding of foreign assets. From (8.41), this means that at $\bar{Q}_0$ the current account will be in overall surplus (i.e., $TB\left(\bar{Q}_0\right) = 0$, and $i^*F > 0 \Longrightarrow CA > 0$). Therefore, to restore equilibrium in the current account, and hence the long-run equilibrium, it requires a further fall in Q, so as to bring about a deficit in the trade balance to offset the surplus in the service account. This occurs at $Q = \bar{Q}_2$ in figure 8.13, where $i^*F = TB(\bar{Q}_2)$ and $CA = 0$.

Observe that the exchange rate overshoots its long-run equilibrium value, as in Dornbusch (1976); but now purely monetary shocks are no longer neutral with respect to the exchange rate, since in the long-run equilibrium the nominal exchange rate $\bar{S}_2$ rises less than prices $\bar{P}_1$, whereas the real exchange rate $\bar{Q}_2$ is below the original value $\bar{Q}_0$. This yields the important result that, in contrast to the Dornbusch model and the monetary approach to the exchange rate, there is also a long-run departure from PPP.

Up to now, the portfolio balance model has been discussed under the simplifying assumption of static expectations, that is, $(dS_t/dt) = 0$. It might be of interest, therefore, to learn what are the implications of the model when expectations of exchange rate change are introduced into the picture. This is done by amending (8.33)-(8.35) to read:

$$M = M\left(i, i^* + \dot{s}^e, W\right), \quad M_i < 0, M_{i^*+\dot{s}^e} < 0, M_W > 0 \quad (8.42)$$
$$B = B\left(i, i^* + \dot{s}^e, W\right), \quad B_i > 0, B_{i^*+\dot{s}^e} < 0, B_W > 0 \quad (8.43)$$
$$SF = F\left(i, i^* + \dot{s}^e, W\right), \quad F_i < 0, F_{i^*+\dot{s}^e} > 0, F_W > 0, \quad (8.44)$$

where $\dot{s}^e \equiv (1/S)\,(dS/dt)$ is the expected depreciation in the exchange rate. As such, the rate of return on foreign assets now takes account of expected changes in the exchange rate, though foreign and domestic assets remain imperfect substitute. Assuming in addition that agents are able to predict exactly the future value of S, the model becomes a perfect foresight model, which is the equivalent of a model with rational expectations in a deterministic world. This means that $\dot{s}^e$ is measuring the actual rate of variation in the exchange rate, that is, $\dot{s}^e = \dot{s}$.

To examine the dynamics of the expectations-augmented model, we need first to rewrite equation (8.41) as

$$\dot{F} = TB(SP^*/P) + i^*F, \quad (8.45)$$

where $\dot{F} = (dF/dt)$ is the change in the country's holdings of foreign assets related to changes in the current account. We then need to solve the model for the two dynamic variables $\dot{s}$ and $\dot{F}$. The behavior of the model is described in figure 8.14. The figure displays in the (F, S) space the two curves defining the equilibrium relationships where $\dot{F} = \dot{s} = 0$, and the paths of F and s away from the long-run equilibrium position or steady state of the system. The $\dot{F} = 0$ locus is downward sloping because a raise in S generates a positive value for $\dot{F}$ through the trade balance, which requires a fall in the interest income through a decline in F, given i^*, to restore equilibrium in the current account. The $\dot{s} = 0$ locus is a rectangular hyperbola, reflecting the fact that, given M and B, asset market equilibrium requires a given value of SF.

The dynamic properties of the model are such that the steady-state equilibrium is a saddle point which can be approached only along the saddlepath AA.

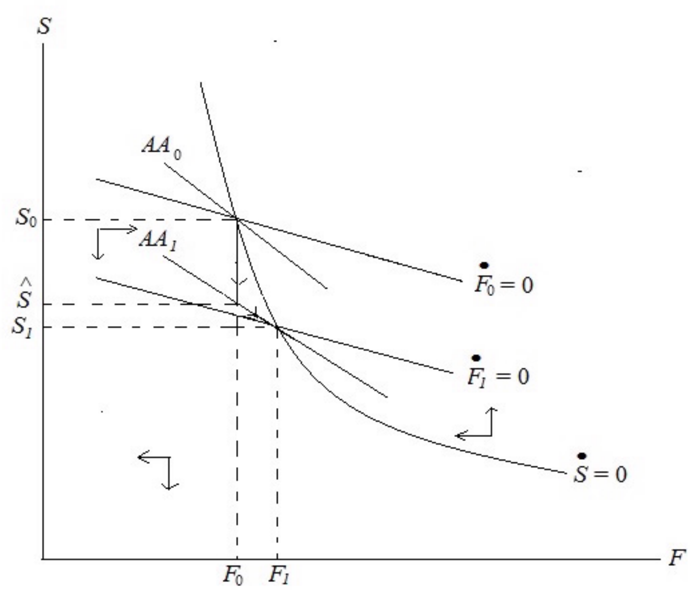

Figure 8.14 Expectations and equilibrium.

8.5 Appendix

A - Solutions to Equation (8.10)

This appendix illustrates the two basic solution methods to the first-order, linear difference equation (8.10): the so-called *backward* and *forward* solutions. To show, let us rewrite equation (8.10) as

$$(1+\alpha_2)\, s_t = [(m_t - m_t^*) - \alpha_1 (y_t - y_t^*)] + \alpha_2 s_{t+1}.$$

Solving for s_{t+1}, yields

$$s_{t+1} = \left(\frac{1+\alpha_2}{\alpha_2}\right) s_t - \frac{1}{\alpha_2} [(m_t - m_t^*) - \alpha_1 (y_t - y_t^*)]. \tag{A.1}$$

Let

$$a \equiv \frac{1+\alpha_2}{\alpha_2} > 1, \;\; (1-a) \equiv -\frac{1}{\alpha_2}, \text{ and } b \equiv [(m_t - m_t^*) - \alpha_1 (y_t - y_t^*)],$$

so that (A.1) becomes

$$s_{t+1} = a s_t + (1-a)\, b_t. \tag{A.2}$$

Backward solution.

To solve backward for s_t, use the recursive nature of (A.2) to obtain, starting at time t, the following sequence of values for s

$$s_t = a s_{t-1} + (1-a)\, b_{t-1} = a\,(a s_{t-2} + (1-a)\, b_{t-2}) + (1-a)\, b_{t-1} \Longrightarrow$$

$$s_t = a^2 s_{t-2} + a\,(1-a)\, b_{t-2} + (1-a)\, b_{t-1} = a^2 [a s_{t-3} + (1-a)\, b_{t-3}] + a\,(1-a)\, b_{t-2} +$$

$$+ (1-a)\, b_{t-1} \Longrightarrow s_t = a^3 s_{t-3} + a^2 (1-a)\, b_{t-3} + a\,(1-a)\, b_{t-2} + (1-a)\, b_{t-1} \Longrightarrow$$

$$s_t = a^n s_{t-n} + a^{n-1} (1-a)\, b_{t-n} + \cdots + a\,(1-a)\, b_{t-2} + (1-a)\, b_{t-1} \Longrightarrow$$

$$s_t = a^n s_{t-n} + (1-a) \left[a^{n-1} b_{t-n} + \cdots + a b_{t-2} + b_{t-1}\right] \Longrightarrow$$

$$s_t = a^n s_{t-n} + (1-a) \sum_{i=t-n}^{t-1} a^{t-1-i} b_i,$$

after n iterations. Letting $n = t$, we obtain

$$s_t = a^t s_0 + (1-a) \sum_{i=0}^{t-1} a^{t-1-i} b_i. \tag{A.3}$$

Equation (A.3) is said to be the *backward solution* of equation (A.2) and expresses s_t in terms of its initial value s_0 and a weighted sum of past values of the (forcing) function b_t.

To find the steady-state solution, or stationary value of s_t, set $s_{t+1} = s_t = \bar{s}$ in equation (A.2), to obtain

$$\bar{s} = a\bar{s} + (1-a)b_t,$$

whence

$$\bar{s} = b_t.$$

Subtracting $\bar{s}$ from both sides of (A.2), we get

$$\begin{aligned} s_{t+1} - \bar{s} &= as_t + (1-a)b_t - \bar{s} = as_t + (1-a)\bar{s} - \bar{s} \Longrightarrow \\ s_{t+1} - \bar{s} &= as_t - a\bar{s} = a(s_t - \bar{s}) \Longrightarrow z_{t+1} = az_t, \end{aligned}$$

where $z_t \equiv s_t - \bar{s}$.

Solving backward for z_t, yields

$$z_t = a^t z_0$$

or

$$s_t = a^t (s_0 - \bar{s}) + \bar{s}, \tag{A.4}$$

where $(s_0 - \bar{s})$ denotes the initial deviation of the system from its steady-state equilibrium $\bar{s}$.

It is easy to find out from(A.4), that the steady-state solution $\bar{s}$ is (globally) asymptotically stable if $|a| < 1$ and unstable if $|a| > 1$, and that the behavior of s_t over time is monotonic or oscillatory according to whether $a \lesseqgtr 0$. Since $a > 1$, we see find that

$$s_t \to \infty \text{ as } t \to \infty,$$

thus showing that the steady-state solution, $\bar{s}$, is asymptotically globally unstable for equation (A.2).

Forward solution.

To solve forward (A.2), rewrite it as

$$as_t = s_{t+1} - (1-a)b_t,$$

whence

$$s_t = \frac{1}{a}s_{t+1} - \frac{(1-a)}{a}b_t,$$

or

$$s_t = \frac{1}{a}s_{t+1} + \frac{1}{1+\alpha_2}b_t, \tag{A.5}$$

since $[(1-a)/a] = -[1/(1+\alpha_2)]$.

Iterating this equation forward for n periods, yields

$$s_t = \left(\frac{1}{a}\right)\left(\frac{1}{a}s_{t+2} + \frac{1}{1+\alpha_2}b_{t+1}\right) + \frac{1}{(1+\alpha_2)}b_t = \left(\frac{1}{a}\right)^2 s_{t+2} + \left(\frac{1}{a}\right)\left(\frac{1}{1+\alpha_2}\right)b_{t+1}$$
$$+\left(\frac{1}{1+\alpha_2}\right)b_t \Longrightarrow s_t = \left(\frac{1}{a}\right)^2\left(\frac{1}{a}s_{t+3} + \frac{1}{1+\alpha_2}b_{t+2}\right) + \left(\frac{1}{a}\right)\left(\frac{1}{1+\alpha_2}\right)b_{t+1}$$
$$+\left(\frac{1}{1+\alpha_2}\right)b_t \Longrightarrow s_t = \left(\frac{1}{a}\right)^3 s_{t+3} + \left(\frac{1}{a}\right)^2\left(\frac{1}{1+\alpha_2}\right)b_{t+2} + \left(\frac{1}{a}\right)\left(\frac{1}{1+\alpha_2}\right)b_{t+1}$$
$$+\left(\frac{1}{1+\alpha_2}\right)b_t \Longrightarrow s_t = \left(\frac{1}{a}\right)^n s_{t+n} + \left(\frac{1}{a}\right)^{n-1}\left(\frac{1}{1+\alpha_2}\right)b_{t+n-1} + \cdots\cdots$$
$$+\left(\frac{1}{a}\right)\left(\frac{1}{1+\alpha_2}\right)b_{t+1} + \left(\frac{1}{1+\alpha_2}\right)b_t \Longrightarrow s_t = \left(\frac{1}{a}\right)^n s_{t+n}$$
$$+\left(\frac{1}{1+\alpha_2}\right)\left[\left(\frac{1}{a}\right)^{n-1}b_{t+n-1} + \cdots\cdots + \left(\frac{1}{a}\right)b_{t+1} + b_t\right] \Longrightarrow$$

$$s_t = \left(\frac{1}{a}\right)^n s_{t+n} + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{t+n-1}\left(\frac{1}{a}\right)^{j-t}b_j,$$

after n iteration. Setting $v = t+n$, $\Longrightarrow n = v-t$, we obtain

$$s_t = \left(\frac{1}{a}\right)^{v-t} s_v + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{v-1}\left(\frac{1}{a}\right)^{j-t}b_j \Longrightarrow$$

$$s_t = a^t\left(\frac{1}{a}\right)^{v} s_v + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{v-1}\left(\frac{1}{a}\right)^{j-t}b_j.$$

Letting $v \to \infty$, yields

$$s_t = a^t\left[\lim_{v\longrightarrow\infty}\left(\frac{1}{a}\right)^{v} s_v\right] + \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{\infty}\left(\frac{1}{a}\right)^{j-t}b_j \Longrightarrow$$

$$s_t = a^t\left[\lim_{v\longrightarrow\infty}\left(\frac{1}{a}\right)^{v} s_v\right] + F_t, \tag{A.6}$$

where

$$F_t \equiv \left(\frac{1}{1+\alpha_2}\right)\sum_{j=t}^{\infty}\left(\frac{1}{a}\right)^{j-t}b_j.$$

From the backward solution (A.3), it follows that

$$\left(\frac{1}{a}\right)^t s_t = s_0 + (1-a)\left(\frac{1}{a}\right)^t\sum_{i=0}^{t-1}a^{t-1-i}b_i \Longrightarrow$$

$$\left(\frac{1}{a}\right)^t s_t = s_0 + (1-a)\left(\frac{1}{a}\right)^t a^{t-1}\sum_{i=0}^{t-1} a^{-i} b_i \Longrightarrow$$

$$\left(\frac{1}{a}\right)^t s_t = s_0 + (1-a)\left(\frac{1}{a}\right)\sum_{i=0}^{t-1} a^{-i} b_i \Longrightarrow$$

$$\left(\frac{1}{a}\right)^t s_t = s_0 + \left(\frac{1-a}{a}\right)\sum_{i=0}^{t-1} a^{-i} b_i \Longrightarrow$$

$$\left(\frac{1}{a}\right)^t s_t = s_0 - \left(\frac{1}{1+\alpha_2}\right)\sum_{i=0}^{t-1} \left(\frac{1}{a}\right)^i b_i.$$

Letting $t \to \infty$, we get

$$\lim_{t\longrightarrow\infty}\left(\frac{1}{a}\right)^t s_t = s_0 - F_0,$$

where

$$F_0 = \left(\frac{1}{1+\alpha_2}\right)\sum_{i=0}^{\infty} \left(\frac{1}{a}\right)^i b_i.$$

Substituting in (A.6), yields

$$s_t = a^t\left(s_0 - F_0\right) + F_t, \tag{A.7}$$

which is the forward solution of equation (8.10).

B - Analytical solution of Dornbusch's Model